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Natural frequencies and mode shapes in zero-force members of a truss

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Abstract. *Trusses are everywhere; they are used in bridges, antenna towers, cranes, even in parts of the International Space Station. And for good reason, they allow us to create strong structures while using materials in very efficient and cost-effective way. Trusses it is essentially a rigid structure made up of a collection of straight members. The type of truss depends on how the horizontal and diagonal beams are arranged.*

Keywords*: trusses, zero-force member, natural frequency, mode shape*

1. Introduction

Trusses are very popular construction by using a relatively small amount of material for the weight they can support [1]. A single-span truss is like a simply supported beam because it carries vertical loads by bending [3]. A truss is a simple structure whose members are subject to axial compression and tension only and but not bending moment [5]. Early trusses were built without precise knowledge of how the loads are carried by each part of the truss. These trusses were patented as from 1840, at a time when new bridge designs we're being developed to accommodate the expansion of the railroad industry and were patented by Howe (fig. 1, a), Pratt (fig. 1, b) and Warren [8].

Warren truss contains a series of isosceles triangles, or equilateral triangles (fig. 2, a). To increase the span length of the truss bridge, verticals are added for Warren Truss (fig. 2, b). Pratt truss is characterized [9] by having its diagonal members (except the end diagonals) slanted down towards the middle of the bridge span.

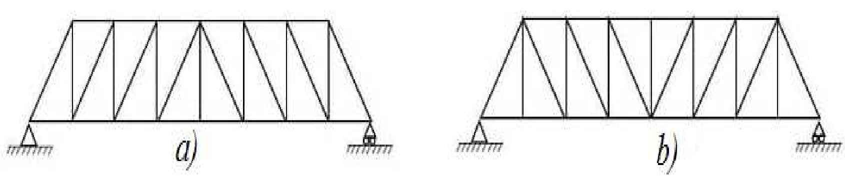

Figure 1. Howe and Pratt truss.

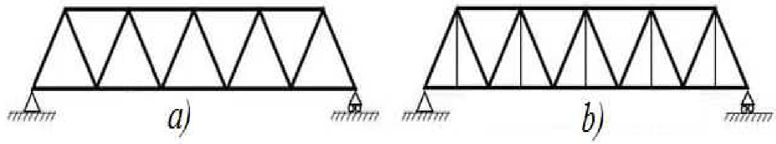

Figure 2. Warren truss.

The members of a truss can be subjected at tension, compression or only participate in increasing the rigidity and stability of the truss without being stressed and the latter are called zero force members. In a truss, the zero force elements are hinged beams at both ends, and their dynamic behavior is analyzed in this paper.

2. Forces in Trusses

To determine the reactions [4] in the points of support the equilibrium equations from the strength of materials are used: the sum of the forces and reactions on the horizontal is equal to zero; the sum of the forces and reactions on the vertical is equal to zero; the sum of the moments of the external forces and of the reactions in relation to a support is zero.

Below is an explanation of what happens if a force is applied to a triangular element

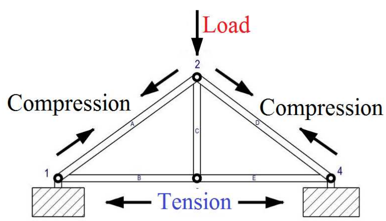

Fig 3 Triangle transfers a force.

When a force (the load) is applied to one of the corners of a triangle, it is distributed down each side. The two sides of the triangle are squeezed. Another word for this squeezing is compression. The third side of the triangle is pulled, or stretched sideways. Another word for this stretching is tension [7].

A simple truss is composed of triangles, which will retain their shape even when removed from supports [6]. A truss is considered statically determinate when the static equilibrium equations can be used to find the reactions on that structure. The method of joints analyzes the force in each member of a truss by breaking the truss down and calculating the forces at each individual joint. When using the method of joints to solve for the forces in truss members, the equilibrium of a joint (pin) is considered.

And by solving the reactions in fig. 4 results which are compressive efforts (negative) and which stretching (positive).

There are rules to identify the beam of a triangle not subject to stresses (compression or stretching), respectively zero-force members:

1. If a joint has only two non-collinear members and there is no external load or support reaction at that joint, then those two members are zero force members;
2. If three members form a truss joint for which two of the members are collinear and there is no external load or reaction at that joint, then the third non-collinear member is a zero force member;
3. Zero-force members can be removed (as shown in the figure 4) when analyzing the truss. The zero-force members are used to increase stability and rigidity of the truss, and to provide support for various different loading conditions.

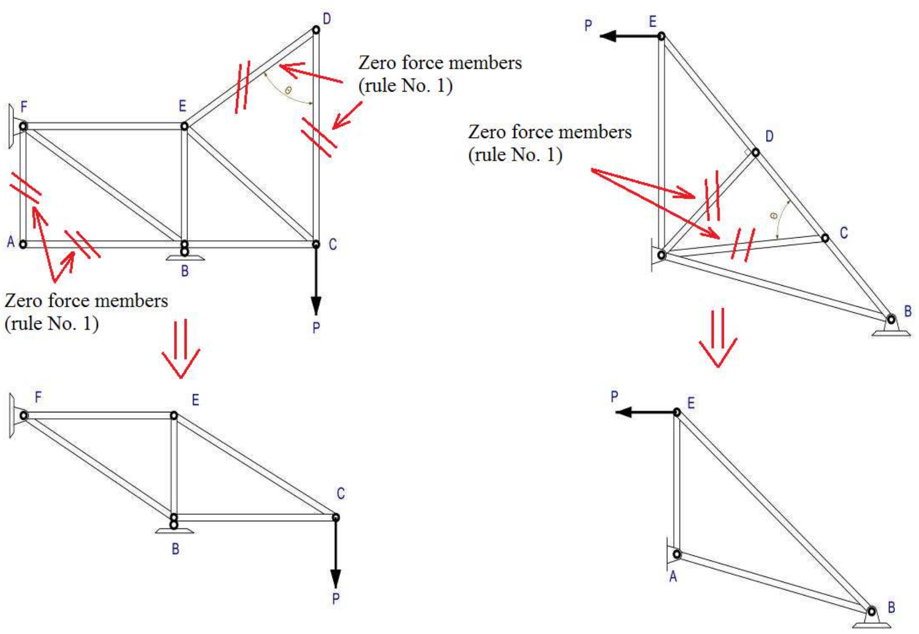

Figure 4. Trusses with zero force members (upper side) and without zero force members (lower side).

3. Natural frequencies and modal shape for a hinged - hinged beam

As shown above, there can be elements with zero forces in a truss. For this element, the natural frequencies and vibration modes can be calculated as a simply supported beam.

For a continuous structure (beam) the frequencies (f_n) are determined by means of the relation (1) [10]:

$$f_n = \frac{a_n^{\ 2}}{2\pi}\sqrt{\frac{E\cdot I}{m\cdot L^4}}\ \text{[Hz]} \tag{1}$$

where, $a_n = \sqrt[4]{\omega^2 \frac{\rho\cdot A}{E\cdot I}}$ (2) is the dimensionless wave number;

ω [rad/s] – is its own pulsation;

ρ [kg/m^3] – the density of the material;

A [m^2] – cross-sectional area;

E [N/m^2] – is the longitudinal modulus of elasticity;

I [m^4] – moment of inertia of the cross section of the beam;

m [kg] - represents the mass of the beam;

L [m] - represents the length of the beam;

$n = 1\ldots\infty$ - vibration mode number.

Knowing the geometry of the structure and the material from which the structure is made, in order to determine the natural frequencies with the relation (1) we must know the values of the dimensionless wave number. In the case of free vibrations, the differential equation of the displacement for transverse vibrations of the beam is:

$$\frac{\partial^4 w}{\partial x^4} - \omega^2 \frac{\rho\cdot A}{E\cdot I_z} w = 0 \tag{3}$$

where, w [m] - vertical movement of the neutral axis. The general solution is:

$$W_n(x) = A_n \sin(a_n x) + B_n \cos(a_n x) + C_n \sinh(a_n x) + D_n \cosh(a_n x) \tag{4}$$

The integration constants A_n, B_n, C_n and D_n are determined from the initial boundary conditions: the deflection and the bending moment at the hinged points are zero, respectively (L is the length of the element):

$$\begin{cases} W(0) = 0 \\ \dfrac{\partial^2 W(0)}{\partial x^2} = 0 \end{cases} \quad (5) \qquad\qquad \begin{cases} W(L) = 0 \\ \dfrac{\partial^2 W(L)}{\partial x^2} = 0 \end{cases} \quad (6)$$

It is obtained such a system of 4 equations with 5 unknowns, which results in the frequency equation:

$$\sin(a_n L) = 0 \tag{7}$$

with solutions: $a_n = \pi, 2\pi, \ldots\ldots n\pi$, respectively the modal function:

$$W_n(x) = \pm \sin(a_n x) \tag{8}$$

The first 6 vibration modes for a simply supported beam of length L = 1 with the normalized function (+ 1) are illustrated in figure 5:

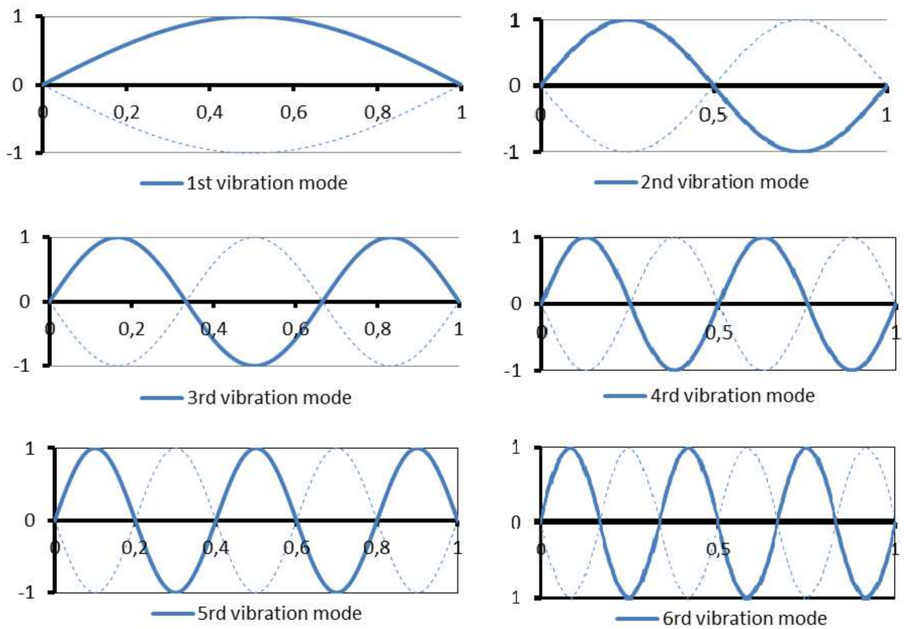

Figure 5. The first 6 vibration modes for simply supported beam.

4. Conclusion

The main conclusions, which highlight the essential elements of the research are:

- if a joint has 3 membres and no external load, or if 2 of the membres are co-liniar, the third non-coliniar member is a zero-force member;

- were put evidence the internal tensions which appears in a triangular element with lattice beams when an external force is applied;

- the rules for zero force in elements of trusses were highlighted;

- for these elements, which are considered as a simply supported beam, the first 6 modes of vibration were drawn.

Knowing the modal function that describes the vibrational movement of the element, we can determine the energy function that allows us to locate the damage for the monitored structure [2].

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