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# Estimation of the severity of damage produced by a transverse crack

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*This paper presents a method to find the severity of a crack for cantilever beams that can be used to estimate the frequency drop due to the crack. The severity is found for the crack located at the location where the biggest curvature (or bending moment) is achieved. Because the fixing condition does not permit a symmetrical deformation around the crack, the apparent severity is smaller as the real one. The latter is found by the estimated value of the trend-line at the fixed end, it being constructed on points that consider the crack position (equidistant points in the proximity of the fixed end) and the resulted deflections.*

***Keywords**: cantilever beam, transverse crack, damage severity, deflection, statistical method*

## 1. Introduction

Knowing the mathematical relation that express the damage severity in respect to the damage depth is essential when the estimation of frequency drop due to cracks is targeted. There are many functions to calculate the damage severity, all being deduced from fracture mechanics. Among the first attempts to express the frequency of damaged beams we mention [1]; further authors focused on finding the severity dependency on crack depth [2] - [5]. Newer research developed this domain [6]. The topic is even the subject of PhD theses, see for instance [7]. However, one of the most reliable relation is given in [8]. In our previous research we approached this topic and found a relation to express the frequency of cracked beams in respect to the damage severity [9] - [11]. The severity is found involving Castigliano's second theorem [12], and it was shown that the severity does not directly depend on the moment of inertia of the damaged cross-section [13].

In this paper we propose a different approach, that involves an energy method, the loss of capacity of the beam to store energy being expressed with help of the free end deflection of the beam under dead load.

## 2. The damage severity

In this section we introduce the damage severity $\gamma(\bar{a})$, which reflects the effect of a crack with depth *a* to a prismatic cantilever beam with width *b* and thickness *h*. The beam has length *L* and the crack is transversal and located at the larger face of the beam. We denoted the relative crack depth $\bar{a} = a/h$. According to [14], the damage severity is calculated as:

$$\gamma(\bar{a}) = \frac{\sqrt{\delta_{(0,a)}} - \sqrt{\delta_U}}{\sqrt{\delta_{(0,a)}}} \tag{1}$$

where $\delta_U$ is the deflection at the free end of the intact beam, and $\delta_{(0,a)}$ is the deflection at the free end of the beam with a crack of depth *a* at the fixed end (namely at distance *x*=0). Because at the fixed end the deformation manifests just on one side of the crack, the severity achieved here is smaller as that expected. To this aim, we simulate cracks at different positions near the fixed end, and find the theoretical severity from the linear regression curve.

The beam has the geometry and mechanical parameters (taken from the ANSYS library for Structural Steel) presented in table 1.

**Table 1.** Beam geometry and mechanical parameters

| *L* (m) | *b* (mm) | *h* | *E* | *ρ* | *ν* |
|---|---|---|---|---|---|
| 1000 | 20 | 5 | $2\text{x}10^{11}$ | 7850 | 0.3 |

The crack positions $x_i$ set in this study are presented in table 2. For all positions, a series of crack depths $a_j$ indicated in table 3 are considered. The width of the crack is always 0.04 mm.

**Table 2.** Positions of the crack relative to the fixed end

| *i* | 0 | 1 | 2 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| $x_i$ (mm) | 6 | 8 | 10 | 12 | 14 | 16 | 18 |

**Table 3.** Depth of the crack

| *j* | 0 | 1 | 2 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $a_i$ (mm) | 0.2 | 0.4 | 0.6 | 0.8 | 1 | 1.2 |
| *j* | 7 | 8 | 9 | 10 | 11 | 12 |
| $a_i$ (mm) | 1.4 | 1.6 | 1.8 | 2 | 2.2 | 2.4 |

## 3. Calculation of the free end deflections

We performed simulation with ANSYS software. The beam was meshed with elements that have the maximum size of the edge 2 mm. For the intact beam, we obtained the deflection $\delta_U = 23.046\,\text{mm}$ at the free end, while for the damage cases the deflections at the free end of the beam are deduced from figures 1 and 2. The trend lines are plotted here and the associated mathematical relations provided together with. Actually, the deflections in these figures are found from the values of the trend line at the distance $x$=0 calculated with the corresponding mathematical relations. Eventually, the estimated deflections for the crack located at the fixed end are given in table 4.

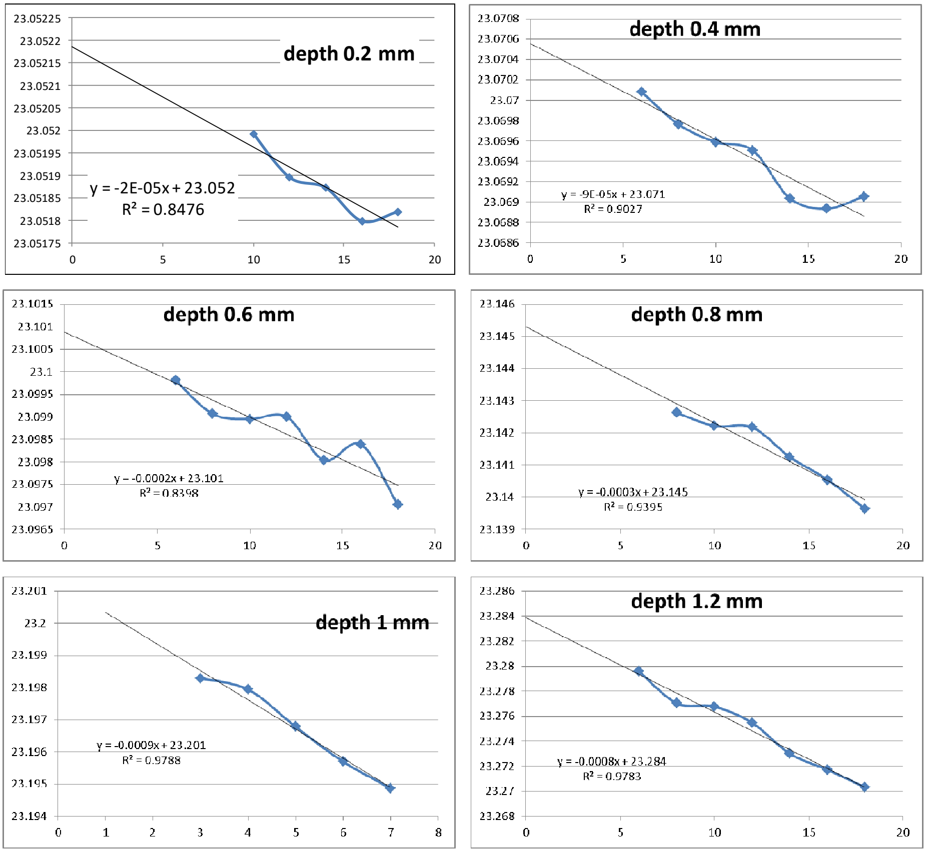


**Figure 1.** Regression curves for the crack depths 0.2 to 1.2 mm

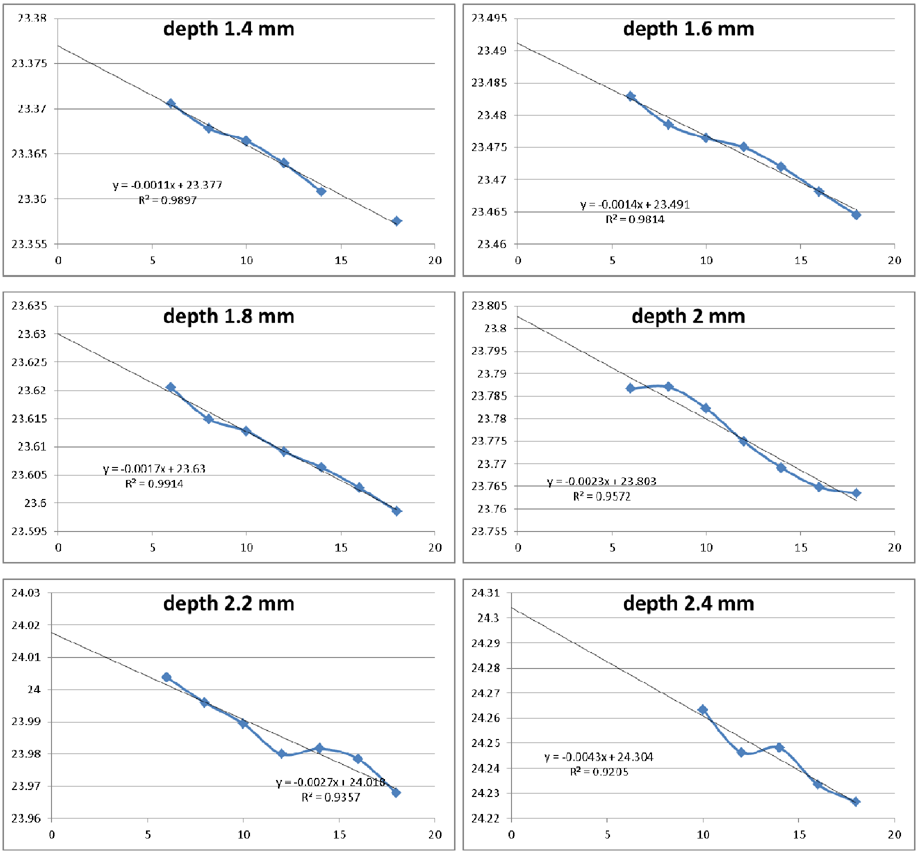


**Figure 2.** Regression curves for the crack depths 0.2 to 1.2 mm

**Table 4.** The deflections found for the considered crack depths

| | | | | | | |
|---|---|---|---|---|---|---|
| $a$ (mm) | 0.2 | 0.4 | 0.6 | 0.8 | 1 | 1.2 |
| $\bar{a}$ (mm) | 0.04 | 0.08 | 0.12 | 0.16 | 0.2 | 0.24 |
| $\delta_{(0,a)}$ (mm) | 23.052 | 23.071 | 23.101 | 23.145 | 23.201 | 23.284 |
| $\gamma(a)$ (mm) | 0.00013 | 0.000542 | 0.001191 | 0.002141 | 0.003346 | 0.005124 |
| $a$ (mm) | 1.4 | 1.6 | 1.8 | 2 | 2.2 | 2.4 |
| $\bar{a}$ (mm) | 0.28 | 0.32 | 0.36 | 0.4 | 0.44 | 0.48 |
| $\delta_{(0,a)}$ (mm) | 23.377 | 23.491 | 23.63 | 23.803 | 24.018 | 24.304 |
| $\gamma(a)$ (mm) | 0.007105 | 0.009517 | 0.012434 | 0.01603 | 0.020444 | 0.026224 |

In table 4, along with the deflections, we show the values ogf the damage severity calculated with relation (1).

### 4. The severity evolution with the damage depth

With the data from table 4 we plot a diagram, figure 3, which represents the evolution of the damage severity with the crack depth. On these points we construct a trend line, as a fourth-order polynomial, and find its mathematical expression. This is given in relation (2).

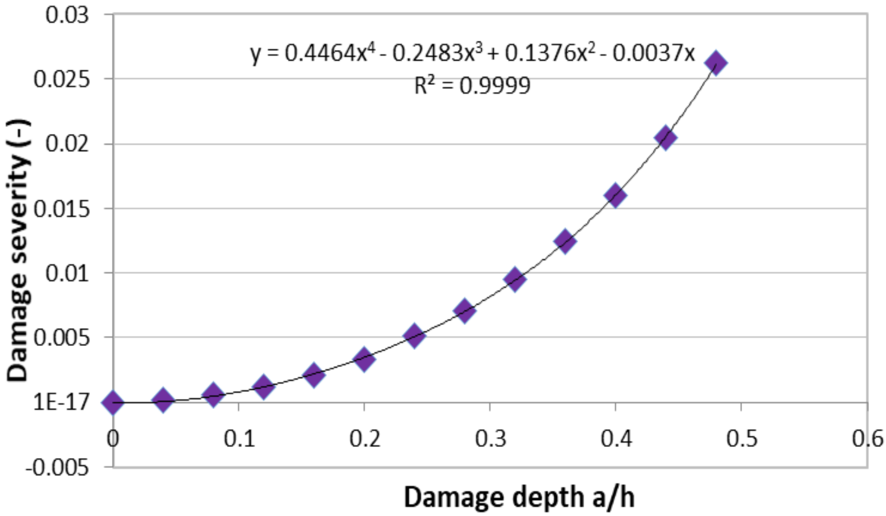


**Figure 3.** The evolution of the damage severity with the crack depth

One can observe that the curve is forced to start from point (0,0), which reflects the real situation; for no damage the deflection increase is inexistent and thus the severity is zero.

$$\gamma(\overline{a}) = 0.4464\overline{a}^4 - 0.2483\overline{a}^3 + 0.1376\overline{a}^2 - 0.0037\overline{a} \qquad (2)$$

A comparison with curves developed by other authors, figure 4, show the reliability of the proposed severity. Note that, the way how the curve is found by the proposed method is simple and fast, dissimilar to the approach of actual methods that use a big number of experiments. In addition, their severity functions depend on the material characteristics, which means that incorrectly determined/involved material characteristics affect the precision of the function.

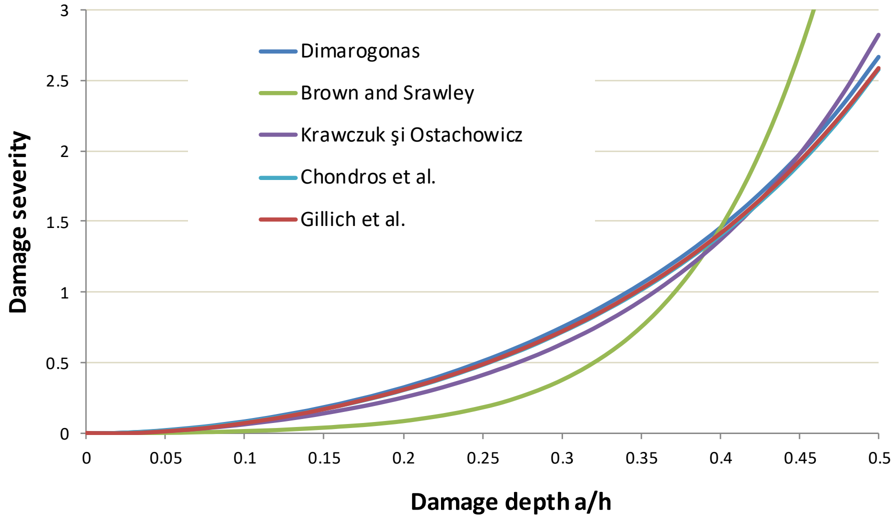


**Figure 4.** Comparison of different damage severity curves

As a second version of the damage severity curve, we plot the severity in function of the thickness ratio of the damaged and undamaged beam, namely $1-\overline{a}$ .

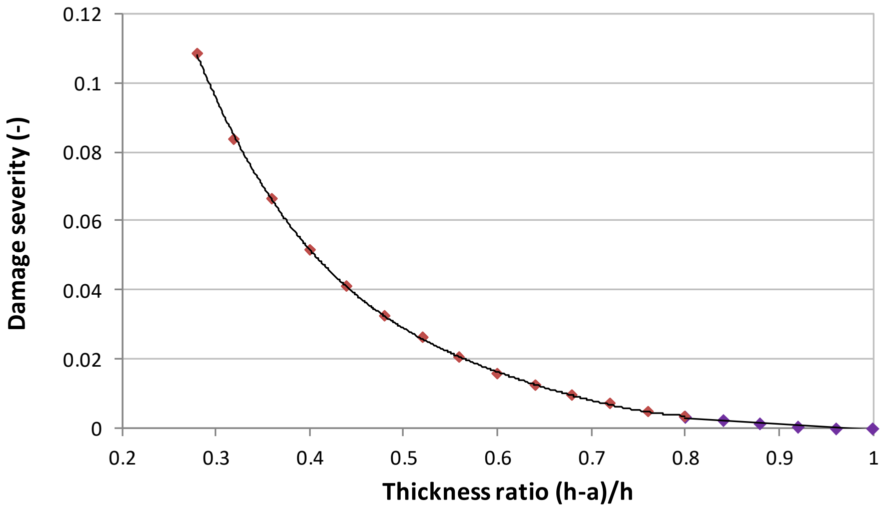


**Figure 5.** The evolution of the damage severity with the thickness ratio

For this curve, one can observe that two intervals for interpolation are necessary, one for a small crack and the second for a deeper crack.

## 4. Conclusion

The paper proposes a mathematical relation to calculate the severity of a crack with help of the deflection of the cantilever beam under dead load deduced for the intact and damaged beam, respectively. The severity increases with the damage depth, the relation being a fourth-order polynomial curve. We also found out that for the curve expressed in respect to the thickness ratio, two intervals of interpolation are requested: one for the incipient cracks and the second for serious damages.

Further research aims to find the damage severity curves for beams that have different cross-section, among which we mention the hexagonal, circular or elliptic shapes. In next research, we will also consider the cases of open and closed cross-sections as I- or T-beams or pipes.

## References


[1] Thomson W.J., Vibration of slender bars with discontinuities in stiffness, *Journal of Applied Mechanics*, 17, 1943, pp. 203-207.

[2] Liebowitz H., Vanderveldt H., Harris DW., Carrying capacity of notched column, *International Journal of Solids and Structures*, 3(4), 1967, pp. 489–500.

[3] Liebowitz H. Claus Jr. W.D.S., Failure of notched columns, *Engineering Fracture Mechanics*, 1(1), 1968, pp. 379–383.

[4] Rizos P.F., Aspragathos N., DimarogonasA.D., Identification of Crack Location and Magnitude in a Cantilever Beam from the Vibration Modes, *Journal of Sound and Vibration*, 138(3), 1990, pp. 381-388.

[5] Dimarogonas A.D., Vibration of cracked structures - A state of the art review, *Engineering Fracture Mechanics*, 5, 1996, pp. 831-857.

[6] Bilello C., *Theoretical and Experimental Investigation on Damaged Beams under Moving Systems,* Ph.D.Thesis, Universita` degli Studi di Palermo, Italy, 2001.

[7] Ostachowicz W., Krawczuk M., *On modelling of structural stiffness loss due to damage*, in DAMAS 2001: 4th Int. Conf. Damage Assessment of Structures, Cardiff, UK, 2001, pp. 185–199

[8] Chondros T.J., Dimarogonas A.D., Yao J., A continuous cracked beam vibration theory, *Journal of Sound and Vibration*, 215(1), 1998, pp. 17–34.

[9] Gillich G.R., Tufoi M., Korka Z.I., Stanciu E., Petrica A., The Relations between Deflection, Stored Energy and Natural Frequencies, with Application in Damage Detection, *Romanian Journal of Acoustics and Vibration*, 13(2), 2016, pp. 87-93.

[10] Praisach Z.I., Gillich G.R., Protocsil C., Muntean F., Evaluation of Crack Depth in Beams for known Damage Location based on Vibration Modes Analysis, *Applied Mechanics and Materials*, 430, 2013, pp. 90-94.
[11] Gillich G.R., Praisach Z.I., Iancu V., Furdui H., Negru I., Natural Frequency Changes due to Severe Corrosion in Metallic Structures, *Strojniški vestnik - Journal of Mechanical Engineering*, 61(12), 2015, pp. 721-730.
[12] Gillich G.R., Maia N., Mituletu I.C., Tufoi M., Iancu V., Korka Z., A new approach for severity estimation of transversal cracks in multi-layered beams, *Latin American Journal of Solids and Structures*, 13(8), 2016, pp. 1526-1544.
[13] Negru I., Praisach Z.I., Gillich G.R., Vasile O., About the Neutral Axis Distortion due to Cracks and its Influence upon the Beams Natural Frequencies, *Romanian Journal of Acoustics and Vibration*, 12(1), 2015, pp. 35-38.
[14] Gillich G.R., Praisach Z.I., Modal identification and damage detection in beam-like structures using the power spectrum and time-frequency analysis, *Signal Processing*, 96, 2014, pp. 29-44.

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