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Determining the optimum severity of damage using model performance analysis methods

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Abstract. *In this work, we have considered a cantilever beam with a rectangular plastic cross-section, reinforced by four wire reinforcements made of ductile Iron. It was studied using numerical resolution based on theoretical formulae and then simulated using Solidworks software. A comparison of the results on the frequency ratios between the theoretical and numerical by Solidworks models gave a low error of 1.389% with a high correlation coefficient of around 0.959. Then, two top reinforcements inside the beam were cut in order to study the behavior of the intact beam and the damaged beam. Using the simulation results, we found an accurate damage severity using a mathematical method based on error deviation indicators or Model Performance Analysis (MPA).*

Keywords: *cantilever beam, frequencies, RMSE, SolidWorks, vibration modes.*

1. Introduction

SOLIDWORKS is a computer-aided design (CAD) software package developed by Dassault Systèmes and used mainly in the mechanical engineering industry. Since its arrival in 1993, this software has revolutionized the engineering world and become the most widely used modeling tool in the industrial sector [1]. Adopted mainly in the automotive and aerospace industries, SolidWorks allows designers to visualize an accurate, to-scale representation of the final product [2] and examine its static and dynamic behavior.

In this work, we propose exploiting the powerful features of the SolidWorks software to study the behavior of a cantilever plastic beam reinforced with four reinforcements inside. The study focuses on the out-of-plain vibrations. The dynamic response of cantilever beams with and without cracks is well-known and comprehensively described in [3]-[5].

The severity of damage is defined in [6], and a mathematical model of beams with cracks is proposed in [7]. In order to obtain reliable results with the model presented by the authors in previous research, it is essential to have a good correlation between the crack depth and the severity.

The contribution of this paper consists of extending the mathematical model's availability for reinforced beams and defining a methodology to find the severity of reinforcement wire damage accurately.

2. Theoretical background and methodology

2.1. About the vibration of beams

We consider the Euler-Bernoulli beam theory, which formulates the problem of the transversely vibrating beam with constant cross-section and rigidity in terms of the partial differential equation of motion [8]:

$$\frac{\partial^4 v}{\partial x^4} + \frac{\rho S}{EI}\frac{\partial^2 v}{\partial t^2} = 0, \tag{1}$$

where v is the vertical displacement of the beam at distance x measured from the clamped end. Considering that v depends on distance x and time t, and the evolution in time is harmonic, the expression of displacement is :

$$v(x,t) = \phi(x).y(t) = \phi(x).\sin(\omega t + \varphi) \tag{2}$$

After derivation of relation (2) and substitution in relation (1), one obtains :

$$\phi^{iv}(x)\sin(\omega t + \varphi) - \frac{\rho S\omega^2}{EI}\phi(x)\sin(\omega t + \varphi) = 0 \tag{3}$$

and, after simplifying by $sin(\omega t+\varphi)$, we have :

$$\frac{d^4\phi(x)}{dx} - \frac{\rho S\omega^2}{EI}\phi(x) = 0 \tag{4}$$

with the well-known solution [8]:

$$\phi(x) = A\sin(kx) + B\cos(kx) + C\sinh(kx) + D\cosh(kx) \tag{5}$$

In the above equation, we denoted:

$$\frac{\rho S\omega^2}{EI} = k^4 \tag{6}$$

The constants A, B, C, and D are determined from the boundary conditions. These derivatives are proportional to the vertical displacement $v(x)$, deflection $\theta(x)$, bending moment $M(x)$, and shear force $T(x)$. Putting the boundary condition for the clamped cantilever beam, given by mechanical reasons :

$$\phi(0) = \phi^{i}(0) = \phi^{ii}(L) = \phi^{iii}(L) = 0 \tag{7}$$

to find the constants A^*, B^*, C^* and D^* we obtain the equation :

$$\cos(\alpha L) * \cosh(\alpha L) + 1 = 0 \tag{8}$$

We can find the n solutions to equation (8) by solving it graphically:

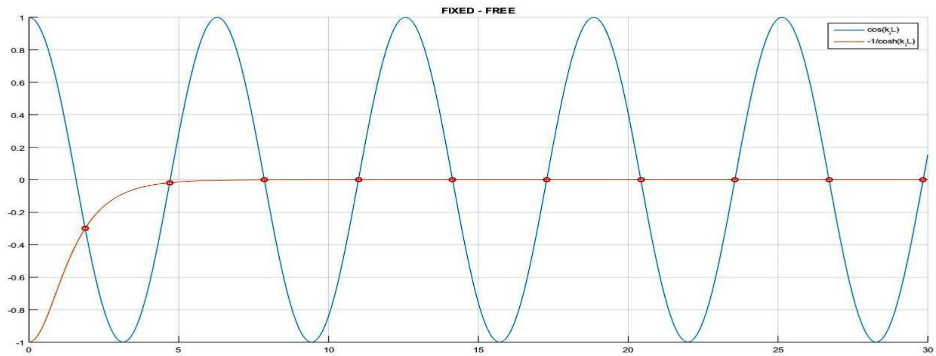


Figure 1. Graphical resolution of the characteristic equation of a cantilever beam

The adimensional wave numbers for the first $n = 6$ modes of vibration of a beam are given in the following table:

Table 1. Wave numbers of a cantilever beam

Modes i	1	2	3	4	5	6
αL	1,8751	4,6941	7,8548	10,9955	14,1372	17,2788

We denote $\lambda = \alpha L$, and calculate the angular frequencies ω_i by multiplying the relation of k^4 by L^4 and substituting the values of λ_i. It results:

$$\omega_i = \frac{\lambda_i^2}{L^2}\sqrt{\frac{EI}{\rho S}}(rad/s) \tag{9}$$

Finally, we find the natural frequencies:

$$f_i = \frac{\omega_i}{2\pi}(hertz) \tag{10}$$

and the periods:

$$T_i = \frac{1}{f_i}(\sec) \tag{11}$$

Obviously, for each angular frequency ω_i there is a corresponding mode shape ϕ_i. The mode shape for the cantilever beam is found, according to [9], with the mathematical relation:

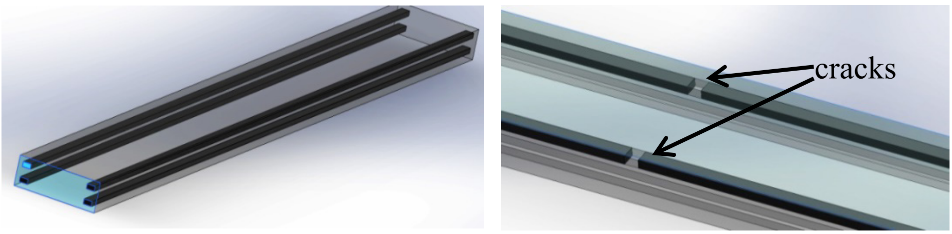
$$\phi(x) = \cosh(\alpha x) - \cos(\alpha x) - \frac{\cos\lambda + \cosh\lambda}{\sin\lambda + \sinh\lambda}\left[\sinh(\alpha x) - \sin(\alpha x)\right] \tag{12}$$

The second derivative of the mode shape represents the mode shape curvature. For a particular bending vibration mode, it can be expressed:

$$\phi''(x) = \cosh(\alpha x) + \cos(\alpha x) - \frac{\cos\lambda + \cosh\lambda}{\sin\lambda + \sinh\lambda}\left[\sinh(\alpha x) + \sin(\alpha x)\right] \tag{13}$$

2.2. The expression of the severity

This section presents a method for determining the deflection at the free end of a cantilever beam with a crack. In our case, the crack consists of two cuttings in the reinforcement wires that are located inside the beam (Figure 2).



a) Intact beam b) Damaged beam

Figure 2. Presentation of the intact beam and damaged beam

The challenge faced when attempting to evaluate damages is that the effect of the crack, both on the deflection and the natural frequencies, is different when it is placed in different positions along the beam [9]. In prior research [7], we have determined a method for assessing the severity of transverse cracks, considering the deflection of a cantilever beam in the intact state and when it is altered by a breathing crack of known depth located at the fixed end. This mathematical relation is:

$$\gamma(a) = 1 - \frac{\sqrt{\delta_U}}{\sqrt{\delta(a,0)}} \tag{14}$$

In Eq.(14), we denoted $\gamma(a)$ the severity of a crack with depth a located at the fixed end; $\delta(a,0)$ the deflection at the free end of the cantilever beam having a crack with depth a at the fixed end (index 0 stays for location $x = 0$ mm); δ_U the deflection of the intact beam at the free end.

In [6], it is presented how to determine the deflection at the free end for the intact beam with a constant cross-section subjected to dead mass:

$$\delta_U = \frac{\rho g S L^4}{8EI}, \tag{15}$$

and the frequency of the undamaged beam becomes:

$$f_{1U} = \frac{\lambda_1^2}{2\pi}\sqrt{\frac{g}{8\delta_U}} \tag{16}$$

Here, ρ is the volumetric mass density, S is the cross-sectional area, g is the gravity, E is Young's modulus, and I is the second moment of inertia.

Similarly, the deflection at the free end for the damaged beam is :

$$\delta_{D\max} = \frac{\rho g S L^4}{8EI_D} \tag{17}$$

and the frequency of the damaged beam [6]:

$$f_{1D} = \frac{\lambda_1^2}{2\pi}\sqrt{\frac{g}{8\delta_{D\max}}} \tag{18}$$

The ratio of the two frequencies gives :

$$\frac{f_{iD}}{f_{iU}} = \frac{\sqrt{\delta_{U\max}}}{\sqrt{\delta_{D\max}}} \tag{19}$$

Consequently, we obtain:

$$f_{iD}(0,a) = f_{iU}\,\frac{\sqrt{\delta_{U\max}}}{\sqrt{\delta_{D\max}(0,a)}} \tag{20}$$

with:

$$\Delta f_{iD}(0,a) = f_{iU} - f_{iD}(0,a) = f_{iU}\left(1-\frac{\sqrt{\delta_{U\max}}}{\sqrt{\delta_{D\max}(0,a)}}\right) \tag{21}$$

The severity is given by:

$$\gamma(a) = \frac{\Delta f_{iD}(0,a)}{f_{iU}} = \frac{f_{iU} - f_{iD}(0,a)}{f_{iU}} = 1 - \frac{f_{iD}(0,a)}{f_{iU}} \tag{22}$$

Thus, we have:

$$f_{iD}(0,a) = f_{iU}\left(1-\gamma(a)\right) \tag{23}$$

The authors [9] established a relationship indicating the change in frequency for any vibration mode *i*, damage depth *a*, location *x*, and any types of bar fixation:

$$f_{iD}(x,a) = f_{iU}\left[1-\gamma(0,a) * \left(\overline{\phi''}(x)\right)^2\right] \tag{24}$$

which makes use of the natural frequency of the intact beam f_{iU}, the damage severity $\gamma(a)$, and the normalized mode shape curvature $\overline{\phi''}(x)$.

2.3. The Model Performance Analysis (MPA) method

This analysis proposes a series of performance indicators to assess the predictive power of a model. The proposed indicators make it possible to evaluate a model's fidelity, exactitude, and accuracy [10].

The bias or Fidelity criteria is the first condition for validating a model, which is that the average of all the deviations e_i should be as close as possible to zero, i.e. an unbiased model. The bias can be calculated as follows:

$$bias = \frac{1}{n}\sum_{i=1}^{n}\left(Y_{real} - Y_{predict}\right) = \frac{1}{n}\sum_{i=1}^{n} e_i \tag{25}$$

The RMSE (Root Mean Square Error), or Exactitude criteria, characterizes the size of the differences between observations and measurements.

The bias indicates differences but does not give us any information about the amplitude of these differences since the positive and negative values of e_i compensate for each other in the mean.

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n} e_i^{\ 2}} \tag{26}$$

The variance of the term e_i over the entire simulation time interval will be defined as the "accuracy" of the model. It can be calculated using the following equation:

$$\sigma_e^2 = RMSE^2 - bias^2 \tag{27}$$

The equations (25), (26), (27) will be used to assess the optimum severity of damaged in order to obtain a high-performance model.

3. Numerical simulations

This work considers a plastic rectangular (6x20 mm²) beam reinforced by four rectangular (1x2 mm²) wires with ductile Iron inside. The length of the cantilever beam is 500 mm.

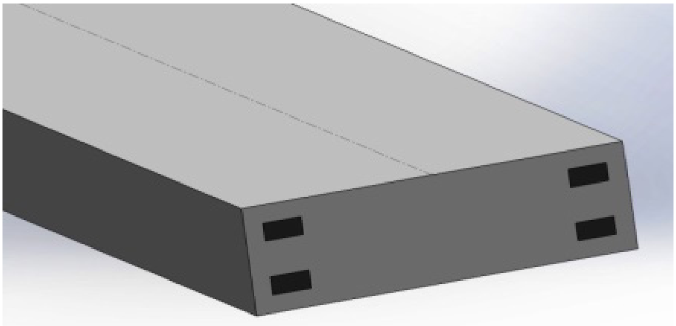


Figure 3. Presentation of the cross-section of the test beam

3.1. Simulation of the intact beam

Using Solidworks with standard parameters, the following table summarises the simulation results for the first six vibration-relevant modes (out-of-plane bending):

Table 2. Frequencies of the intact beam for the first six vibration modes

Modes	1	2	3	4	5	6
ω_i *(rad/sec)*	58.227	363.45	1011.3	1964	3209.9	4730.9
f_i *(Hz)*	9.267	57.844	160.95	312.58	510.87	752.95
T_i *(sec)*	0.10791	0.017288	0.006213	0.003199	0.001957	0.001328

The following figure shows the stresses for the first four vibration modes for the intact beam :

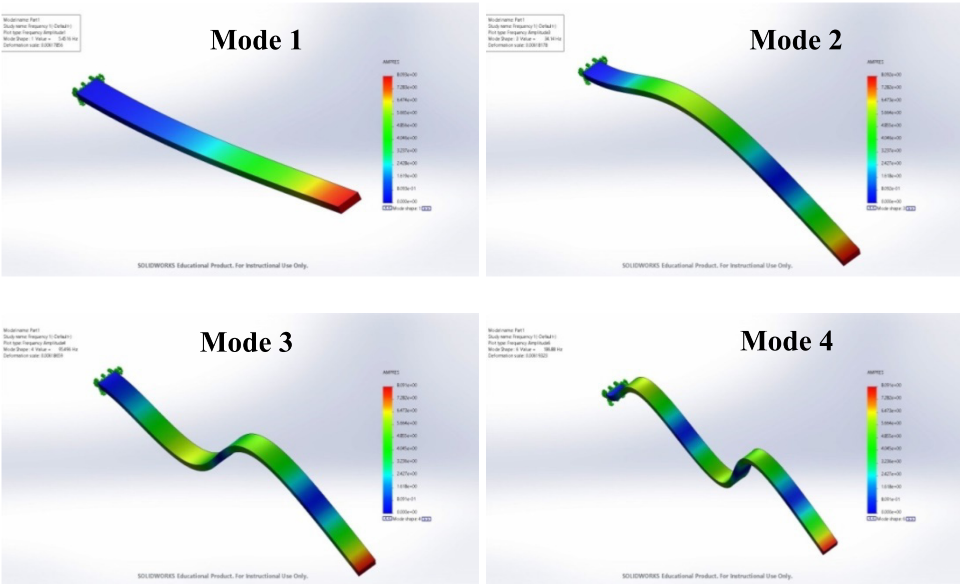


Figure 4. Modes shape of the intact beam using Solidworks

3.2. Simulation of the damaged beam

Now, we cut two of the reinforcements inside the beam. The width of the cut is 2 mm, and *X[mm]* is the location from the fixed end of the beam. Using Solidworks with standard parameters, the following table summarises the simulation results for the first four vibration-relevant modes:

Table 2. Frequencies of the damaged beam for the first four vibration modes

X (mm)	Mode 1	Mode 2	Mode 3	Mode 4
1	9.091	56.844	158.41	308.14
6	9.091	56.883	158.6	308.64
20	9.146	57.305	159.9	311.25
60	9.162	57.672	160.95	312.44
100	9.178	57.848	160.6	310.63
140	9.192	57.79	159.74	311.17
180	9.204	57.515	159.66	312.55
220	9.219	57.124	160.55	309.84
260	9.238	56.956	160.85	308.5
300	9.256	57.142	159.82	311.86
340	9.266	57.414	159	311.96
380	9.271	57.642	159.25	309.38
420	9.274	57.797	160.28	310.04
460	9.276	57.866	160.9	312.15
494	9.279	57.912	161.12	312.89
499	9.279	57.916	161.15	312.96

Figure 5 shows the curves given by Solidworks (blue) from Table 2 compared with the theoretical (red) for the first four modes of vibrations.

After cutting the reinforcement of the beam, we confirm that the results found on Solidworks with the damaged beam allow us to have a curve shape that is well synchronized with the theoretical but with errors that oscillate between [0 : 0.0050], which is accepted.

Table 3. Error indicator values for each vibration mode

Modes	1	2	3	4
MAE	0.00126	0.00184	0.00132	0.00146
RMSE	0.00168803	0.00239106	0.00157144	0.00197598
Correlation coefficient	0.9784	0.9437	0.9731	0.9409

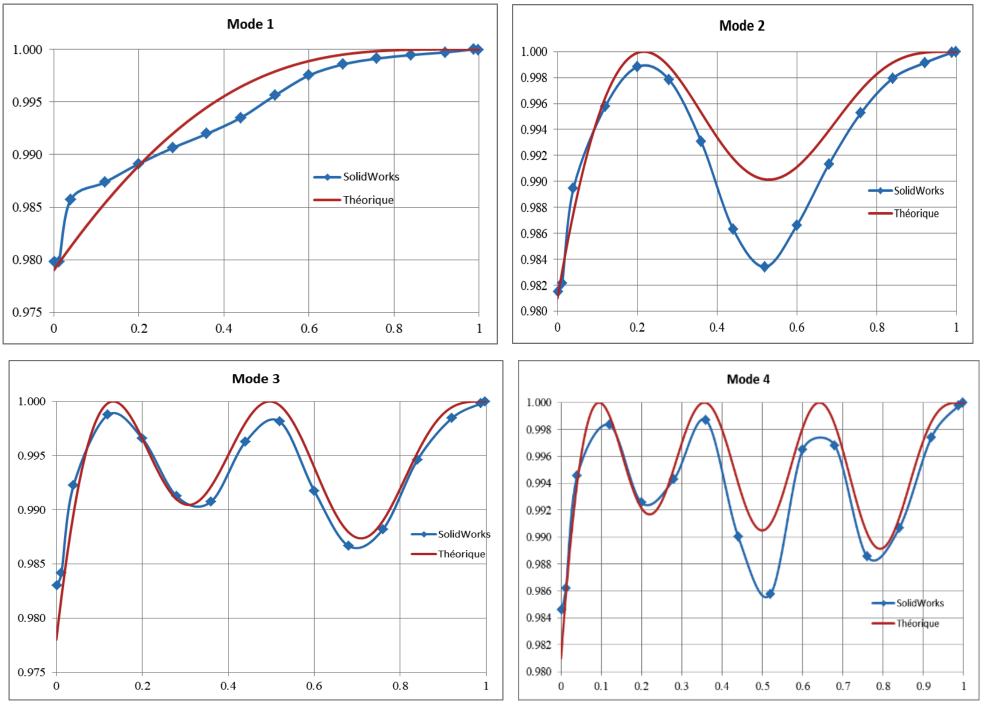


Figure 5. Frequencies of the damaged beam plotted with results from Solidworks (blue line) and calculus (red line)

We conclude that for each vibration mode, the values of the error deviation indicators are acceptable. Solidworks can generate the theoretical results well, with an average correlation coefficient of 0.9590, which is satisfactory.

3.3. Severity assessment using the MPA method

Figure 6 shows that the values of the indicators (MAE and RMSE) change as a function of the damage severity coefficient. Therefore, we propose the following approach to determine the optimum value for severity:

- plotting a points cloud of MAE and RMSE as a function of the severity coefficient,
- the optimal coefficient corresponds to the minimum value of MAE (Mean Absolute Effor) and RMSE (Root Mean Square Error) together.

The following example was made using vibration mode number 3:

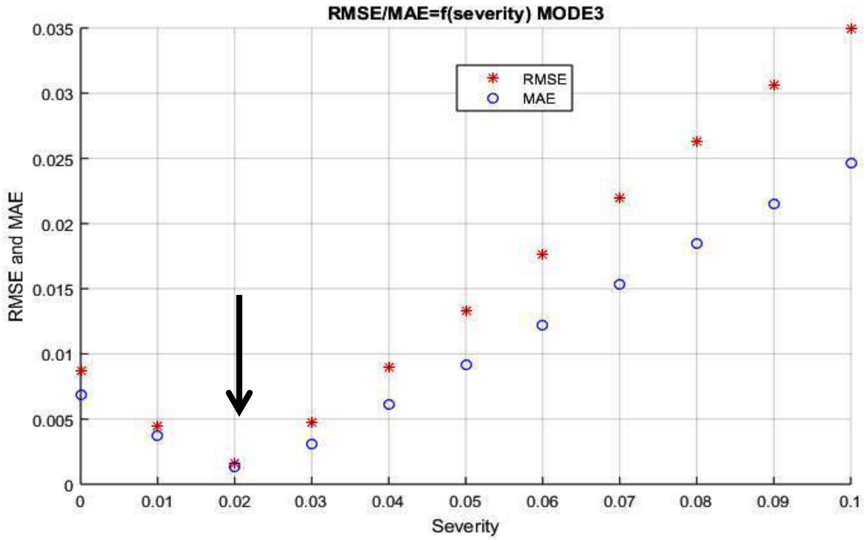


Figure 6. Points cloud for determining optimum severity

We deduce that for mode number 3, the optimum severity is 0.020. We found the optimum severity values for each vibration mode using the same procedure. The results are presented in Table 4.

Table 4. Optimum severity values for each vibration mode

Modes	Mode 1	Mode 2	Mode 3	Mode 4
Optimum severity	0.020	0.022	0.020	0.019

From the values in Table 4, we estimate the optimum severity as the average of the severities for the considered modes, which is 0.02025. We chose a single value for the severity for all modes since this was demonstrated in [6].

4. Conclusion

We used SolidWorks to accurately model and analyze the behavior of a reinforced cantilever beam subjected to vibration. The simulations showed perfect synchronization between the theoretical vibrations of the beam and the SolidWorks simulation results. Thus, we confirmed the reliability of the models developed in SolidWorks for structural dynamics applications.

We also determined the severity value for the considered damage using a probabilistic approach based on error indicators, and the results are demonstrated as very satisfactory.

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