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THE GEOMETRIC SYSTEM OF THE NAVE VAULT OF THE CHURCH ON THE HILL OF SIGHIŞOARA

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ABSTRACT. The Geometric System of the Nave Vault of the Church on the Hill of Sighişoara. The literature on the methods of late Gothic net vault constructions mainly accentuates ideas, which presume that the initial conditions of the design are orderly. However, in the case of buildings of multiple building periods, this is rarely the case. In the present article, we present the point cloud-based geometric analysis of the net vault system of the Church on the Hill of Sighişoara. Our results highlight that contrary to the generally accepted theories, the construction of a net vault is not necessarily initiated with the plan of the rib system, and that the underlying principle organising the whole system's geometry is not always the plan pattern. Our case study also exemplifies how the Gothic principles of unity and division still blend with the ideas of separation and addition, even in a late Gothic hall church with direct connections to the guild of Landshut.

Keywords: *masonry ribbed net vault; late Gothic vault construction; geometric analysis; Church on the Hill of Sighişoara; Gothic construction principles*

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1. Introduction

The technical literature on the topic of late Gothic net vault construction generally accentuates the same widespread ideas, such as the primacy of the plan of the net vaults' rib pattern in the construction process, or the "Prinzipalbogen" principle. However, such ideas work ideally only in cases when the initial conditions, like the borderline of the examined vaults' plan, are regular.

The subject of the present article, the three-aisle nave vault system of the Church on the Hill of Sighișoara is an excellent example of the fact, that numerous vaults, especially those of buildings of multiple building periods, have no such baseline circumstances (Fig. 1). The major theories do not give explanations on how these factors modify the orderly theories. However, by getting to know the exact geometry of a net vault system, the geometric deviances can be analysed and the regularities in the geometry identified.

In the present article, our aim is to conduct such an analysis, based on the method we developed during our previous works, and to describe the underlying ideas uniting the seemingly chaotic vault system.

Fig. 1. The three different net vaults of the nave. (right: northern aisle, middle: main nave, left: southern aisle). Photos by Krisztina Fehér PhD

1.1. Literature framework

The technical literature of the geometry of late Gothic net vaults is hallmarked by several deeply rooted ideas that were mainly spread by the 19th century's influential works on the topic of Medieval vaults.

The first important ascertainment is that these theories without exception accentuate the primacy of the construction of the vault's plan, even though the next steps may differ. Sources representing this idea appear from the 16th century (e. g. Hontañon's drawing[1] and the Manuscript Ms. 12686 of the National Library of Spain[2]), followed by the works of Ranisch,[3] Hoffstadt,[4] Warth,[5] or Ungewitter.[6] Examples of the exact methods can be the construction based on a quadrate-net's crossing points (e. g. Schulze's method),[7] or an original square's further division with inscribable rotated quadrats (e. g. Hoffstadt's XIV. A. board).[8] The theory's practical aspect is that according to recent research, the real-size construction of the rib pattern on the building site could have served as the initial step of the building process.[9] To conclude, we want to accentuate that the presentations of these theories always assume a regular plan, thus the irregularities, which often characterise the buildings of multiple building periods are not taken into account.

The next necessary step based on the technical literature is to ascertain the vertical dimensions of the vaults. In this regard, the sources show a more diverse picture. The main theories use hemi-cylindric (basically barrel vault-like) – or, in the case of stellar vaults, hemispheric – surfaces to describe the spatial

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1 Santiago Huerta, "Technical Challenges in the Construction of Gothic Vaults: The Gothic Theory of Structural Design," in *Bautechnik des Historismus. Von den Theorien über gotische Konstruktionen bis zu den Baustellen des 19. Jahrhunderts*, ed. Uta Hassler and Christoph Rauhut (München: Hirmer Verlag, 2012).

2 Ricardo Garcia Baño, and Macarena Salcedo Glera, "Geometry and Construction of the Eight-Loop Ribbed Vault," *Nexus Network Journal* 22, no. 4 (2020).

3 Bartel Ranisch, *Beschreibung aller Kirchengebäude der Stadt Dantzig…* (Dantzig: Raths und Gymnasii Buchdruckern, 1695).

4 Friedrich Hoffstadt, *Gothisches ABC-Buch: Vorlegeblätter zum gothischen A-B-C-Buche…* (Frankfurt a. M.: Siegmund Schmerber, 1840).

5 Otto Warth, *Die Konstruktionen in Stein* (Leipzig: J. M. Gebhardt, 1896).

6 Georg Gottlob Ungewitter, *Lehrbuch der gotischen Konstruktionen. Neue bearbeitet von K. Mohrmann* (Leipzig: Chr. Herm. Tauchnitz, 1901).

7 Werner Müller, "Die Zeichnungsvorlagen für Friedrich Hoffstadts 'Gotisches A.B.C.-Buch' und der Nachlass des Nürnberger Ratsbaumeisters Wolf Jacob Stromer (1561-1614)," *Wiener Jahrbuch für Kunstgeschichte* 28, no. 1 (1975): 45.

8 Müller, "Die Zeichnungsvorlagen," 52.

9 David Wendland, and Frédéric Degenève, "How to Order Fitting Components for Looping Ribs: Design Procedures for the Stone Members of Complex Late Gothic Vaults," in *Building Histories: The Proceedings of the Fourth Conference of the Construction History Society*, ed. James W.P. Campbell et al. (Cambridge: Queens' College, 2017), 164.

positions of the ribs' junction points,[10] even though practical considerations, such as economical aspects and the required workload, suggest that these descriptions are rather the abstract, simplifying ideas behind a linear temporary supporting structure. This more practical aspect of the idea is presented on Hontañon's drawing, where the keystones are elevated to the required height (determined by a hemisphere) individually, thus presenting the structure behind the idea.

The general geometry of the net vaults and the geometry of their individual ribs are nearly always examined simultaneously in the sources of the topic. The majority of the researchers rely on the "Prinzipalbogen" theory, meaning that each rib in a given net (or stellar) vault has the same curvature (thus accelerating the building process).[11] The idea appears in the technical literature from the 16th century on (e.g. the manuscript of Jacob von Andernach[12], and in the works of Ranisch,[13] Hoffstadt,[14] Meckel,[15] Müller,[16] Tomlow[17]), and although some researchers gave voice to doubts (e. g. Lassaulx in 1835),[18] it remained the leading theory even until today, and in certain cases, the most recent studies also approved it.[19] The method to determine the value of the rib curvature's radius varies in the above-cited sources, some claim that it equals

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10 Ranisch, *Beschreibung*; Warth, *Die Konstruktionen*; Ungewitter, *Lehrbuch.*

11 Jürgen Renn, Wilhelm Osthues, and Hermann Schlimme, *Wissensgeschichte der Architektur 3. Vom Mittelalter bis zur frühe Neuzeit* (Berlin: Edition Open Access, 2014), 71; R. Maira Vidal, "The Evolution of the Knowledge of Geometry in Early Gothic Construction: the Development of the Sexpartite Vault in Europe," *International Journal of Architectural Heritage* 11, no. 7 (2017): 1007.

12 Werner Müller, "Einflüsse der österreichischen und der böhmisch-sächsischen Spätgotik in den Gewölbemustern des Jacob Facht von Andernach," *Wiener Jahrbuch für Kunstgeschichte* 27, no. 1 (1974): 65–66.

13 Ranisch, *Beschreibung.*

14 Hoffstadt, *Gothisches.*

15 Carl Anton Meckel, "Figurierte Gewölbe der deutschen Spätgotik," *Architectura: Jahrbuch für Geschichte der Baukunst* 1 (1933).

16 Werner Müller, *Grundlagen gotischer Bautechnik. Ars sine scientia nihil* (München: Deutscher Kunstverlag, 1990).

17 Jos Tomlow, "Versuch einer (zeichnerischen) Rekonstruktion des Gewölbes im spätgotischen Kreuzgang des Klosters Hirsau," in *Hirsau St. Peter und Paul 1091–1991*, ed. Klaus Schreiner (Stuttgart: Landesamt für Denkmalpflege im Regierungspräsidium Stuttgart, 1991).

18 David Wendland, "Johann Claudius von Lassaulx' Gewölbe >aus freier Hand< – Die Wiedererfindung der gotischen Architektur und die Entwicklung der technischen Literatur," in *Bautechnik des Historismus. Von den Theorien über gotische Konstruktionen bis zu den Baustellen des 19. Jahrhunderts*, ed. Uta Hassler, and Christoph Rauhut (München: Hirmer, 2012), 106.

19 Clemens Voigts, "Bauforschung an figurierten Gewölben der Spätgotik: Das Beispiel der Georgskirche in Augsburg," *Architectura – Die Zeitschrift für Geschichte der Baukunst / Journal of the History of Architecture* 45 (2015): 56–57; Jobbik, Eszter and János Krähling, "A Self-contained Stellar Vault Construction Method. The Vault of the Matthias Oratorio in the Inner City Parish Church of Budapest," *Periodica Polytechnica Architecture*, Online first publication (2023).

half of the diagonal described by the given rib's plan,[20] while others refer to the "principle of the longest route", meaning that the radius of the curvature is the longest continuous rib-route from the impost to the crown point on the plan (16th-century sketchbook from Dresden[21] and the works of Hoffstadt 1840[22], Ungewitter[23] and Meckel[24]). According to some researchers (e. g. Müller),[25] using ribs with uniform curvature only served the purpose of three-dimensional positioning of the ribs' junction points.

As for the webbing of the net vaults, according to the commonly accepted theory, it was built posterior to the rib system, using it as centring (as described by Saunders in 1814 and specified later by Willis in 1842,[26] either with formwork,[27] resulting in a nearly flat web-surface,[28] or without it as the individual courses form self-supporting arches during the building process.[29] Even in the case of masonry ribbed vaults, this building technique was described by certain researchers.[30] However, several works claim that in the case of ceramic ribs, the webbing is in fact a barrel vault built anterior to the rib system, the latter serving only decorative purposes.[31]

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[20] Ranisch, *Beschreibung*; Elena Pliego, "Georg Gottlob Ungewitters Lehrbuch der gotischen Constructionen," in *Bautechnik des Historismus. Von den Theorien über gotische Konstruktionen bis zu den Baustellen des 19. Jahrhunderts*, ed. Uta Hassler, and Christoph Rauhut (München: Hirmer, 2012), 407.

[21] François Bucher, "Medieval Architectural Design Methods, 800–1560," *Gesta* 11, no. 2 (1972): 47.

[22] Hoffstadt, *Gothisches*, XIV.A/5.

[23] Ungewitter, *Lehrbuch.*

[24] Meckel, "Figurierte".

[25] Olaf Huth, *Entwurfs- und Konstruktionsprinzipien des spätgotischen Netzgewölbes der Kirche St. Peter und Paul in der Lutherstadt Eisleben* (Bamberg: University of Bamberg Press, 2020), 27.

[26] David Wendland, "Traditional Vault Construction without Formwork: Masonry Pattern and Vault Shape in the Historical Technical Literature and in Experimental Studies," *International Journal of Architectural Heritage: Conservation, Analysis, and Restoration* 1, no. 4 (2007): 342.

[27] Clemens Voigts, "Vaults, Centring, and Formwork of the Late Gothic Period in Southern Germany," in *History of Construction Cultures,* vol. 2, ed. João Mascarenhas-Mateus, and Ana Paula Pires (Boca Raton: CRC Press, 2021), 78.

[28] Wendland, "Traditional," 342.

[29] Voigts, "Vaults," 79.

[30] Szőke Balázs, "A Wechselberger-Harperger motívum Délkelet-Erdély késő gótikus építészetében," in *Tanulmányok a székelység középkori és fejedelemség kori történetéből*, ed. Sófalvi András, and Visy Zsolt (Énlaka: Pro Énlaka Alapítvány and Haáz Rezső Múzeum, 2012), 207–208; Szőke Balázs, "Téglabordás boltozatok Dél-Erdélyben," in *Colligite fragmenta! Örökségvédelem Erdélyben,* ed. N. Kis Tímea (Budapest: ELTE BTK, 2009), 72–73.

[31] Victor Roth, *Geschichte der deutschen Baukunst in Siebenbürgen* (Strassburg: Heitz&Mündel, 1905), 36; Hermann Fabini, *Atlas der siebenbürgisch-sächsischen Kirchenburgen und Dorfkirchen* (Hermannstadt: Monumenta Verlag Hermannstadt and Arbeitskreis für Siebenbürgische Landeskunde e.V. Heidelberg, 1999); Harsányi István, "A szeged-alsóvárosi ferences templom gótikus szentélye csillagboltozatának helyreállítása," *Műemlékvédelem* 45, no. 5 (2001): 302.

1.2. The history of the vault system

The Church on the Hill of Sighişoara is a result of numerous building periods. The subject of the present article, the net vault system of its nave was built during the great Gothic rebuilding of the church's western part in the 15th century, from 1429 to 1525,[32] which became necessary due to damages caused by the anomalies in the load-bearing capacity of the hill's ground.[33] During these works, the asymmetric, irregular wall contours of the Romanesque period were not changed. Thus, the deviance in the angle of the western tower's walls to those of the nave and the tower's east-west axis, which is shifted compared to that of the nave,[34] must be considered as given circumstances regarding the construction of the nave vault. The inner space was altered by building only three pairs of columns instead of the four Romanesque ones and creating a hall church instead of a basilica.[35]

After an earthquake in 1838, reconstruction works were needed.[36] The vault of the choir was destroyed and later rebuilt out of wood.[37] However, based on the written sources, it is unclear whether parts of the nave's vault system were concerned. Several sources mention the damage to the aisles' or the main nave's vaults[38] as well, although the extent of the damage is not detailed.

Based on our geometric research, the overall geometry of the rib system is highly consequent, even though it seems chaotic at the first glance (details below), thus we claim that the nave's vault system could not have been largely concerned by the earthquake, since neither of the sources refer to a significant rebuilding phase. However, some tumbled ribs are plausible, but in ribbed

32 Roth, *Geschichte*, 66.

33 George Oprescu, *Die Wehrkirchen in Siebenbürgen* (Dresden: Sachsenverlag Dresden, 1961), 55; Daniela Marcu Istrate, "Voraussetzungen und Vorbedingungen für den Bau der Bergkirche in Schäßburg," *Zeitschrift für Siebenbürgische Landeskunde* 41 (2018): 10; Daniela Marcu Istrate, "Entstehung und Entwicklung der siebenbürgischen Stadtkirchen im 12-15. Jahrhundert," in *Sachgeschichte(n). Beiträge zu einer interdisziplinär verstandenen Archäologie des Mittelalters und der Neuzeit. Festschrift für Barbara Scholkmann zu ihrem 80. Geburtstag*, ed. Dorothee Ade et al. (Tübingen: Tübingen Library Publishing, 2021), 456–457.

34 Marcu Istrate, "Voraussetzungen," 13; Marcu Istrate, "Entstehung," 457.

35 Marcu Istrate, "Voraussetzungen," 14.

36 Friedrich Müller, "Die Schässburger Bergkirche in Siebenbürgen," *Mittheilungen der K. K. Central-Commission* 1 (1856): 172.

37 Christoph Machat, *Die Bergkirche zu Schässburg und die mittelalterliche Baukunst in Siebenbürgen* (München: Verlag des Südostdeutschen Kulturwerks, 1977), 62; Christoph Machat, ed., *Denkmaltopographie Siebenbürgen. Stadt Schäßburg*, 4.1 (Köln: Rheinland-Verlag GmbH, 2002), 97; Marcu Istrate, "Voraussetzungen," 5.

38 Roth, *Geschichte*, 67.

vaults, in such cases, the realignment of the inner forces[39] can result in another equilibrium state,[40] and the system can remain stable, as numerous examples show, thus we claim that the nave vault likely kept its late Gothic form.

2. Methodology

As we stated above, our geometric analysis is based on the laser-scanned point cloud of the church. The point cloud-based analysis of different vaults and vault systems has already appeared in the technical literature, however, the systematic, reproducible approach we use in our research projects was developed by us. Huth[41] used a scan-generated point cloud for measuring rib curvatures and keystone heights, Bianchini[42] examined the exact geometry of domes by scanning, Vidal[43] also measured the exact geometry in his research about rib deformations, while Fuentes and Huerta[44] and Gonzalo and Talaverano[45] worked with vault geometries based on laser total station and laser distance meters.

As the first step of our research, we use Leica BLK 360 space scanner and Leica Cyclone Register 360 software to create the point cloud of the building (Fig. 2). The further steps are carried out in AutoCAD software. During our geometric analysis, we lay great emphasis on differentiating the geometry connected to the previous building periods, thus meaning a given circumstance during the construction of the examined vaults and the geometry of the parts constructed in the analysed building period. Therefore, the first step of our analysis is to find the period borders, on the one hand, based on written sources, and in the absence of those, based on the building geometry itself.

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[39] Gábor Lengyel, and Katalin Bagi, "Numerical Analysis of the Mechanical Role of the Ribs in Groin Vaults," *Computers and Structures* 158 (2015): 58.

[40] Jacques Heyman, *The Stone Skeleton. Structural Engineering of Masonry Architecture* (Cambridge: Cambridge University Press, 1995), 20–22; Huerta, "Technical," 183.

[41] Huth, *Entwurfs.*

[42] Carlo Bianchini, "A Methodological Approach for the Study of Domes," *Nexus Network Journal* 22, no. 4 (2020).

[43] Vidal, "The evolution".

[44] Paula Fuentes, and Santiago Huerta, "Geometry, Construction and Structural Analysis of the Crossed-arched Vault of the Chapel of Villaviciosa, in the Mosque of Cordóba," *International Journal of Architectural Heritage* 10, no. 5 (2017).

[45] J. C. Palacios Gonzalo, and R. Martín Talaverano, "Technological Development in Spanish Gothic Vaults Design," *International Journal of Architectural Heritage* 7, no. 2 (2013).

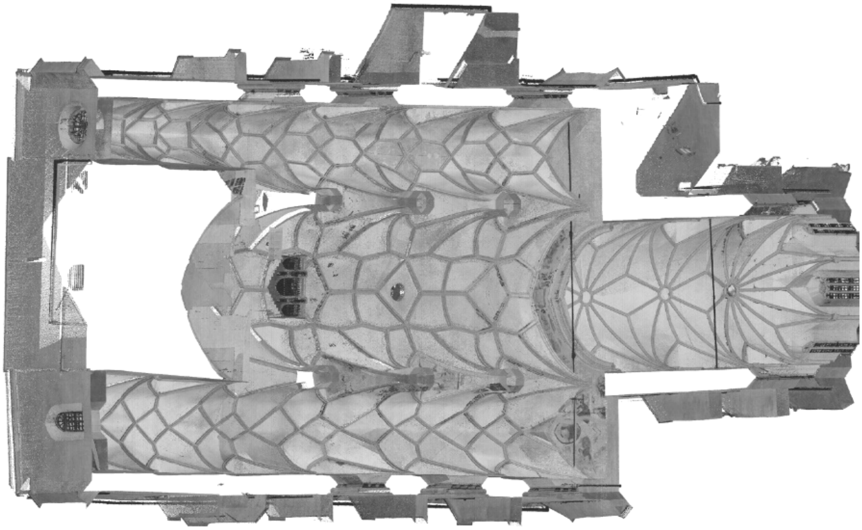

Fig. 2. The picture of the vault's point cloud. The authors' work

Since the vault presented in this article has masonry ribs, we start our analysis by examining the connection of the ribs to the webbing. For this, the point cloud gets cut by horizontal planes thus creating the "mapping" of the system. If the lines describing the webbing are parallel to the walls and do not correspond to the position of the ribs, the main structure is proved to be a barrel vault, whereas if the lines are not straight and reflect the rib system's geometry, the ribs were built posterior to the webbing and served as its formwork. This geometric description of the structures' connection is a simple one, however, its examination only became possible due to the laser scanning technologies.

Then, we analyse the plan of the net vault, while looking for regularities suggesting an underlying construction idea of this view of the rib system – which, contrary to the widespread ideas of the technical literature, was not used in each case.[46] The next step is the analysis of the rib junction points spatial position, by examining the interdependence of their projected pictures to the longitudinal and cross-sections. The subject of our further analysis is the geometry

46 As proved in Eszter Jobbik, and János Krähling, "Late Mediaeval Net Vault Construction Method Rediscovered by Geometric Analysis. A Case Study of the Fortified Church of Băgaciu (Bogeschdorf)," *Brukenthal. Acta Musei* 17, no. 2 (2022).

of the individual ribs of the given rib system, thus the numeric values of the ribs' radius of the curvature, chord length and arch length. By finding the regular features in the rib system's geometry, the original ideas behind the construction of the structure can be presumed.

3. Results

As we detailed above, it is crucial to differentiate the given circumstances and the conscious decisions regarding the net vault's construction. Based on the church's building history (see above), we know, that the position of the walls of the former building period determined the Gothic wall contours as well, thus the "contour line" of the vault system must be interpreted as a condition to which the design of the vault must have been adjusted. Although the number of the nave's pillars was reduced during the Gothic building period, the excavation showed that the disposition of the Romanesque pillars influenced the position of the Gothic ones.[47]

3.1. The connection of the webbing to the rib system

As we described in the Methodology chapter, the connection of the webbing to the rib system was carried out by creating the "mapping" of the net vault (Fig. 3). The level lines clearly indicate that, with the exception of the webs adjoining the crown line, the webs were built on the ribs. However, right next to the crown line, the webs show more barrel vault-like features – although they are still not straight. Based on our previous research, changing the masonry technique where the surface tangential to the webbing gets close to horizontal is not an unprecedented solution (e.g. the stone ribbed apse vault of Andocs).[48] Thus, we found that the nave vault of the Church on the Hill of Sighişoara is not a barrel vault with decorative ribs installed on it, but a real net vault with the webbing built on the rib system as a formwork.

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47 Marcu Istrate, "Voraussetzungen," 8.

48 Eszter Jobbik, and János Krähling, "Remodelling a Medieval Net Vault Construction. Case Study: the Apsis Vault in the Catholic Church of Andocs," *Építés-Építészettudomány* 50, no. 3–4 (2022).

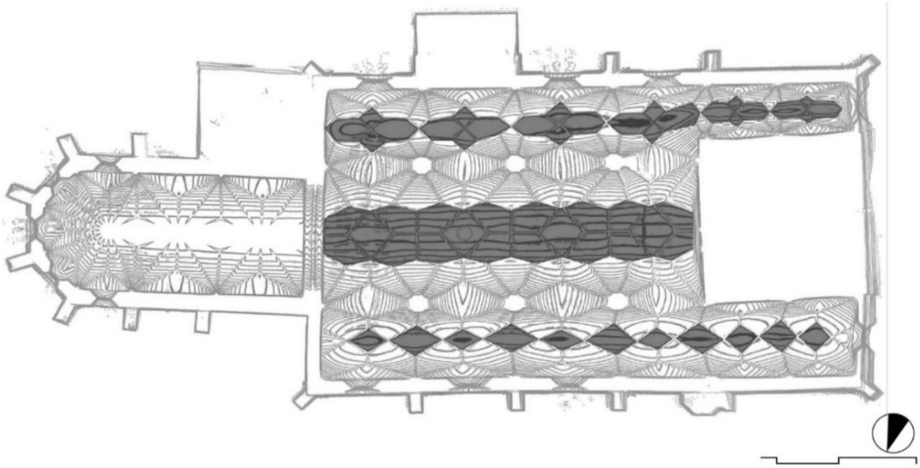

Fig. 3. The mapping of the vault. The grey marks the barrel vault-like parts close to the crown line. The authors' work

3.2. Plan analysis

For the analysis of the vault systems' plan, we were looking for recognisable regular lines and circles (representing the even distances from a given surface or point) on which the correspondent junction point's pictures fall (Fig. 4, 5, 6, 7). Regarding these figures, it is clear even at first glance that the two aisles' rib plans are not regular enough to consider them constructed, since even in the easternmost bays, a number of junction points deviate from the lines which could be presumed as the basis of their positions. However, the middle nave's rib pattern could be accepted as actually constructed on the plan view. Nonetheless, the latter theory must be rejected due to the main nave's westernmost bay, where the main lines describe the position of some of the junctions, but the position of several other junction points can not be constructed based on the same principles as the three western bays. Considering this latter finding as well as the more irregular plan geometry of the aisles, which originated from the same building period as the main nave, we concluded, that the plans of the rib systems were likely not the base of the whole system's construction. (We presume that the more regular features of the main nave are due to the fact that its net vault is supported by the columns of the nave, which are from the Gothic building period, and although they are not distributed entirely evenly, they eliminate some of the irregularities of the walls. Also, in the case of the middle nave, the problem to solve the more irregularly shaped spaces' vaults next to the tower did not occur, thus we assume that the construction of the simpler situation was adjusted to that of the more difficult one, and not the other way around.). However, the longitudinal lines, on which the junction points of the same function in the system fall, can be consistently found throughout all three naves (Fig. 8). (Even though other lines and circles could be also studied as the base of the aisle's construction, we claim that if our basic examinations did not highlight a regular construction idea, it is very likely that others would not lead to a different conclusion either.)

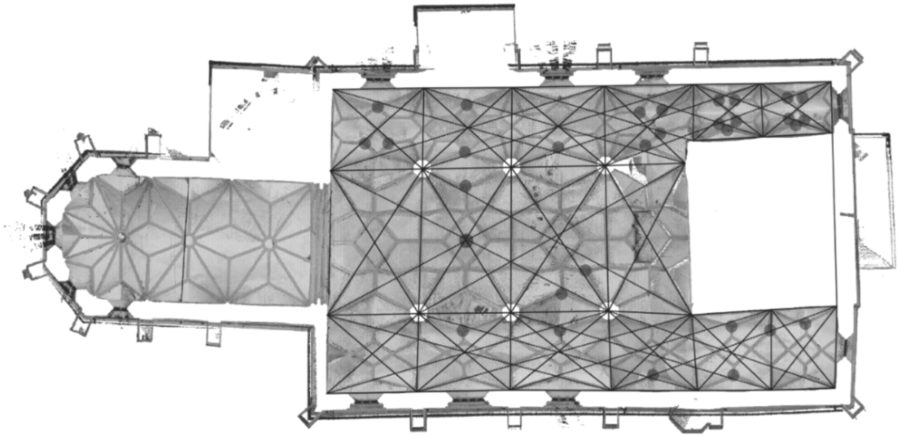

Fig. 4. The examination of the vault system's plan. Analysing the lines between the imposts. The dots mark the junctions which do not lay on the line closest to its position. The authors' work

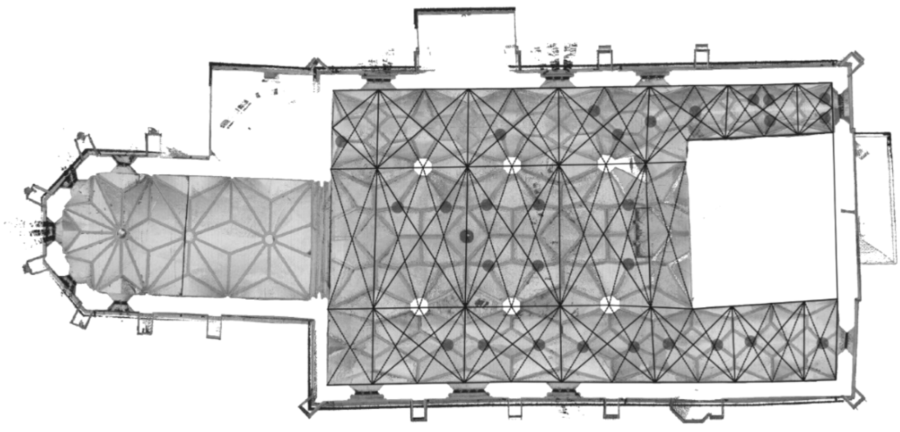

Fig. 5. The examination of the vault system's plan. Analysing the lines between the midpoints of the bays' sides. The dots mark the junctions which do not lay on the line closest to its position. The authors' work

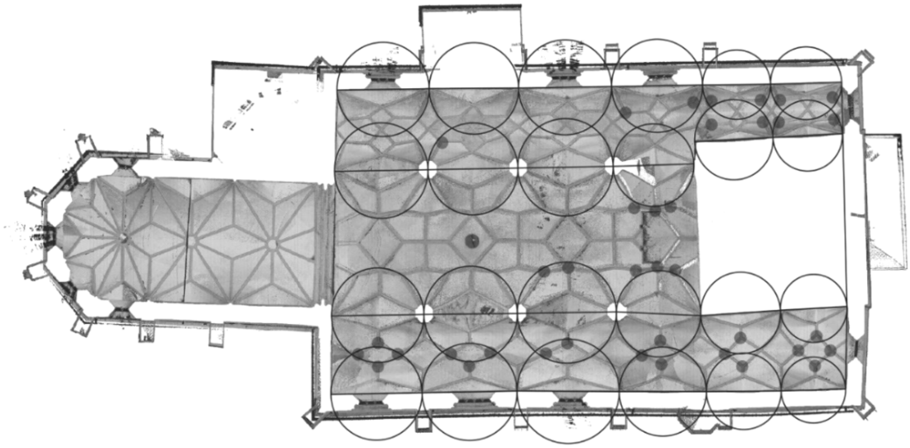

Fig. 6. The examination of the vault system's plan. Analysing the Thales' circles drawn on the bays' sides. The dots mark the junctions which do not lay on the line closest to its position. The authors' work

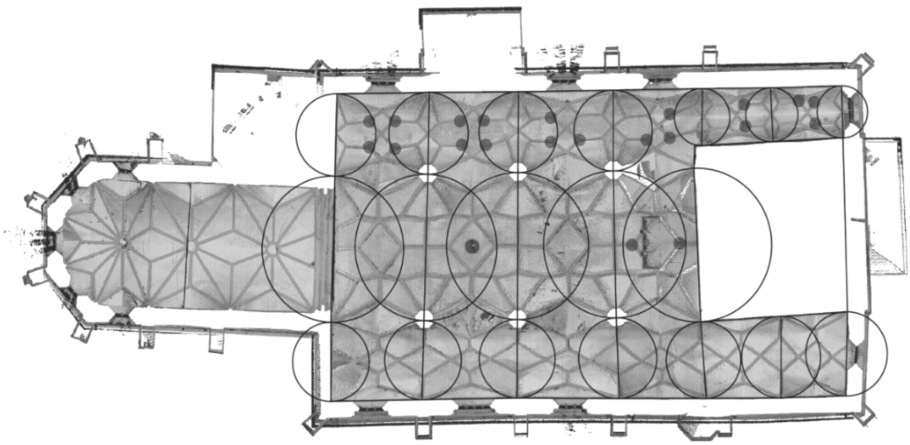

Fig. 7. The examination of the vault system's plan. Analysing the Thales' circles drawn on the lines dividing the bays from each other. The dots mark the junctions which do not lay on the line closest to its position. The authors' work

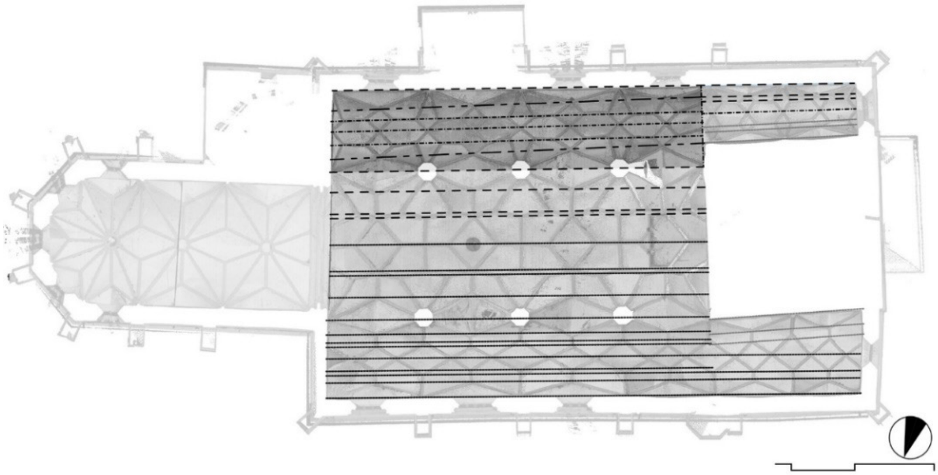

Fig. 8. The examination of the vault system's plan. The longitudinal lines determining the junction points' position on the plan view. The authors' work

In the case of the northern aisle, the lines representing the plan's regular feature are parallel to the northern nave wall, with the exception of the southern side of the westernmost two bays of the vault, where the direction of the tower's northern wall determines them. The longitudinal lines of the main nave also follow the direction of the northern wall on the northern side of the wall, as well as the main axis, while on the southern side of the vault, the lines are parallel to the southern wall.

However, in the case of the southern aisle, the lines are not parallel to the southern wall, as it would ensue from the ascertainments described so far. Here, the plan of the vault can be clearly divided into an eastern and western part, the latter being significantly narrower than the former due to the asymmetry of the western tower's placement. The axes of the two parts are not coinciding, although the construction idea is the same behind both: they connect the middle points of the short sides of their borderlines. In the case of the western part of the southern aisle, the same basic logic applies as in the case of the western part of the northern aisle: the lines of the northern side are parallel to the tower's southern wall, while those of the southern side are parallel to the southern wall of the nave. In the case of the eastern part, two different logic applies. On the one hand, the southernmost line connects the endpoint of the correspondent line of the western part and a point constructed on the cross-section of the vault projected to the eastern short wall (detailed below). The northernmost line is parallel to the southernmost one, and one of its endpoints is the southeastern corner of the tower. On the other hand, the lines closer to the crown line are parallel to the axis of this part of the vault.[49]

3.3. The analysis of the longitudinal sections

The analysis of the longitudinal sections showed, in the case of all three naves, that the corresponding junction points fall – approximately – to the same line, as well as that these lines are parallel to each other throughout the whole net vault system (Fig. 9ab, 10ab, 11ab).

Fig. 9a. The examination of the vault system's longitudinal sections. The longitudinal sections of the northern aisle looking to the north. The authors' work

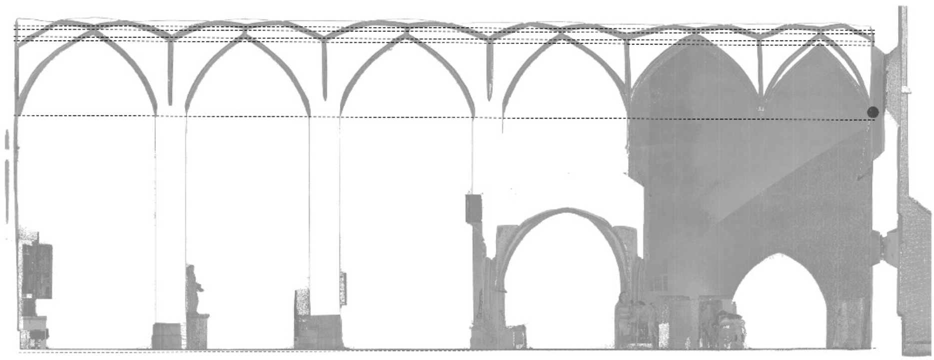

Fig. 9b. The examination of the vault system's longitudinal sections. The longitudinal sections of the northern aisle looking to the south. The authors' work

[49] The positions of the junction points, which do not fall to the lines on which the majority of the points of the same function fall, is to be explained at the last paragraph of chapter 3.5.

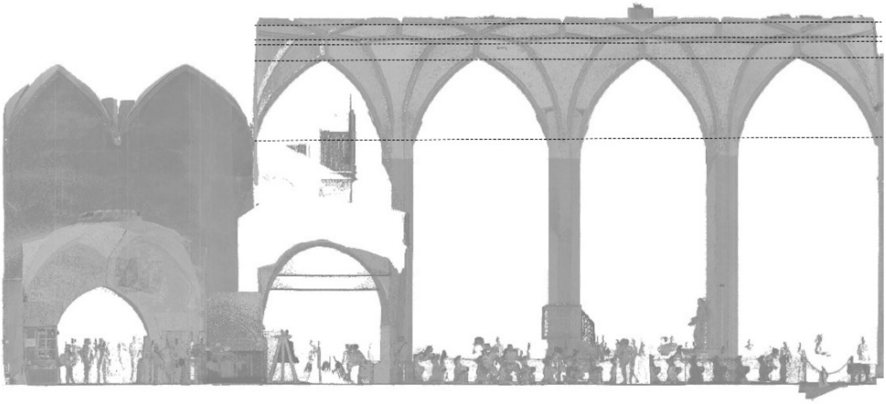

Fig. 10a. The examination of the vault system's longitudinal sections. The longitudinal sections of the main nave looking to the north. The authors' work

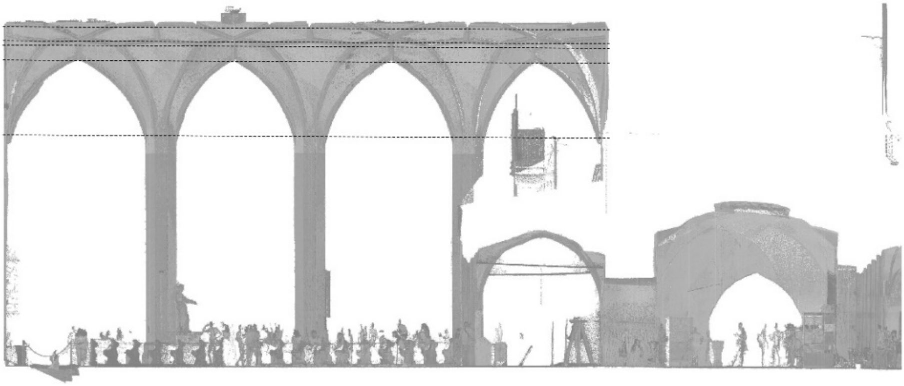

Fig. 10b. The examination of the vault system's longitudinal sections. The longitudinal sections of the main nave looking to the south. The authors' work

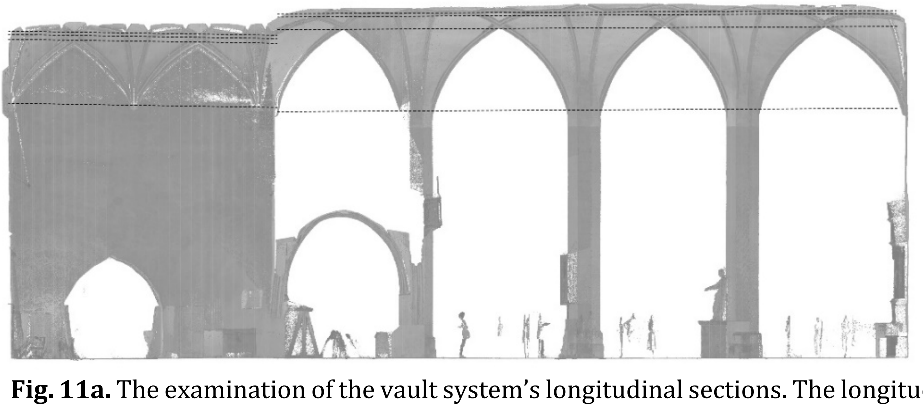

Fig. 11a. The examination of the vault system's longitudinal sections. The longitudinal sections of the southern aisle looking to the north. The authors' work

Fig. 11b. The examination of the vault system's longitudinal sections. The longitudinal sections of the southern aisle looking to the south. The authors' work

3.4. The analysis of the cross-sections

The analysis of the cross-sections was carried out by analysing the position of the points where the lines found on the longitudinal sections intersect the eastern short wall of the nave. Since the analysis of the plan and longitudinal sections suggested its necessity, we accomplished the same examination with the western short wall of the southern aisle as well. (It is to be noted that in the case of the southern aisle, the height of the net vault also changed significantly at the borderline between the two parts differentiated on the plan view, whereas in the case of the northern aisle, only a slight direction change of the lines was needed to adjust to the given geometry, thus in the latter case this analysis was found pointless.)

Regarding the eastern short wall, we were able to reconstruct a quadrate net as the base for the construction of the vault. The net of the two aisles has a 4:3, while the main nave has a 6:4 width-to-height ratio. Even though the quadrates of the aisles are smaller than those of the main nave, their dimensions are geometrically cohering since the side of the smaller quadrates is three-fourths of that of the bigger ones (Fig. 12).

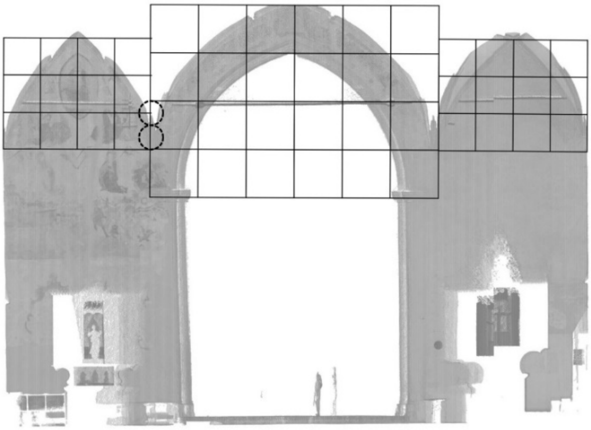

Fig. 12. The underlying quadrate nets describing the three naves' geometry. The authors' work

We found that the lines found on the plan and the longitudinal sections of the vault system are also organised by these quadrate nets. In the case of the two aisles, the determined heights are coinciding, even though the plan patterns of the two vaults differ significantly, and they can be deduced based on the eight of the quadrate's side lengths, whereas in the case of the main nave the sixth of the quadrate's side length leads to the vertical positions of the junction points. As for the distances of the lines of the plan, the same ratios gave the solution (Fig. 13).

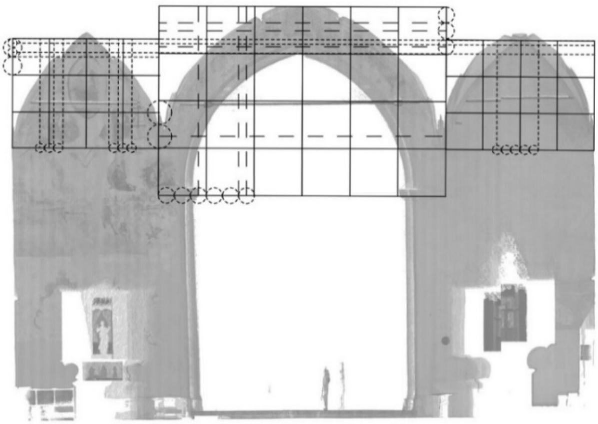

Fig. 13. The underlying quadrate nets describing the three naves' geometry with the ratios describing the different junction types' height values (on the eastern short walls). The authors' work

As for the western short wall of the southern aisle, a square can be drawn as the base of the construction, the side of which is equal to two and a half units of the quadrate net of the aisle's western side. The heights of the lines found on the longitudinal section can be deduced from the eight of the side of the eastern short wall's quadrate, like in the case of the eastern part of this aisle (Fig. 14).

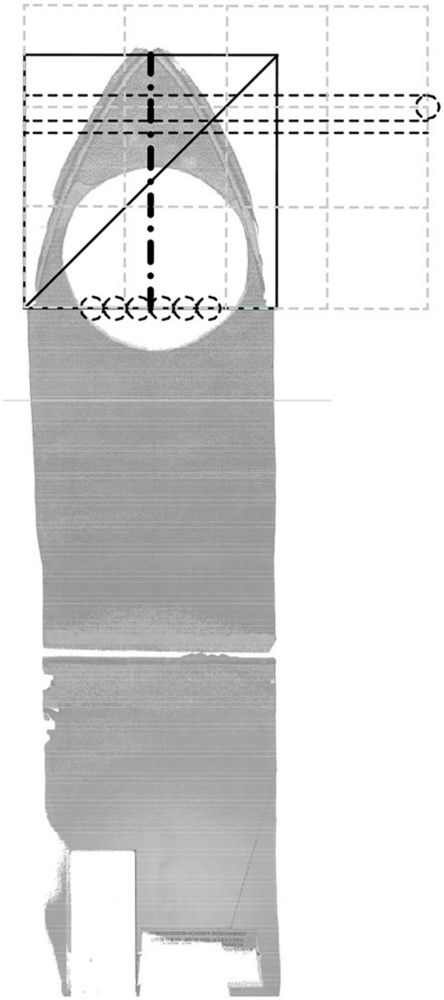

Fig. 14. The underlying quadrate nets describing the southern nave's geometry with the ratios describing the different junction types' height values (on the western short wall). The authors' work

3.5. The analysis of the individual ribs

It is to be accentuated that the number of regular features found during the analysis of the rib system's global geometry so far is not enough to unequivocally construct the spatial position of a given junction point, thus some additional data is needed. We found this data in the characteristics of the individual ribs. We measured the radius of the curvature and the chord length of each rib in the system. (Table 1. and Table 2. The legend for Table 1. appears on Fig. 15).

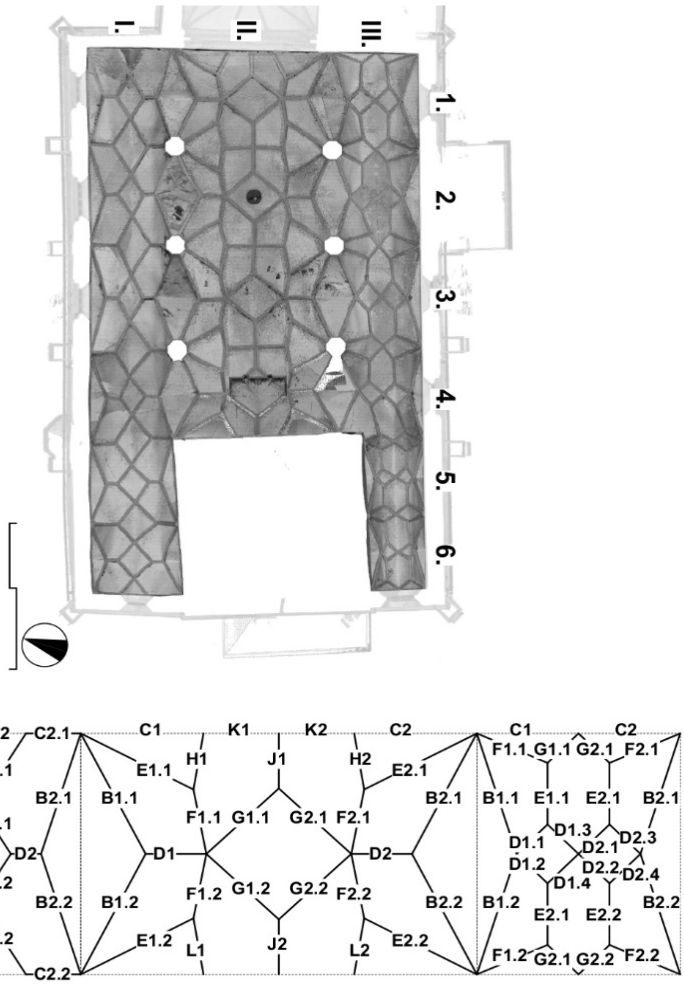

Fig. 15. Legend for the individual ribs' alphanumeric signs. The authors' work

As for the above-detailed "Prinzipalbogen" principle, we concluded that the values of the radius of the curvature show a significant deviation, thus can not be accepted as uniform. Consequently, the principle does not apply to the nave vault system of the Church on the Hill of Sighişoara.[50]

However, regarding the chord length values of the ribs, we found them highly consequent with the ribs' position in the system, even though the ribs' lengths projected to the plan may differ significantly. Thus, we claim that the chord length of the ribs is the data which completes the construction of the rib system of the net vaults (Fig. 16). (It is to be noted that in the cases where the values are not consistent, the exceptions are always those ribs which constitute the vaults' connections to the imposts. Since the sizes of the bays are not coincidental – not even the easternmost ones, where the tower does not disturb the system – these border positions provide sufficient explanation for the anomalies.)

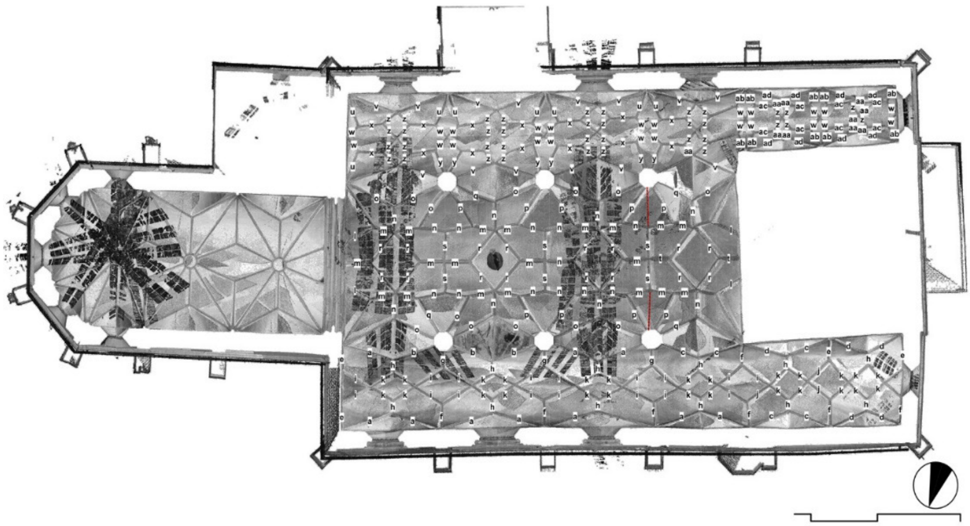

Fig. 16. The distribution of the chord length types signed on the plan. The authors' work

It must be mentioned, that while the majority of the junction points' plan pictures actually fall on the lines described in this chapter, there are certain exceptions. Most of the exceptions are only slightly displaced, which can be accepted as a result of the movement of the temporary supporting structure during the building works, or the builders' effort to fit the prefabricated rib

[50] It is to be noted that even though measuring the curvature of a rib is not an easy task, as Vidal (Vidal, "The Evolution," 1009) proved by experiential method, regardless of the deterioration, the curvature of the ribs can be found with a high probability.

elements in the designated spaces between the junction points. However, in the case of the southern aisle, some junctions' deviation from the lines on which the others of the same function fall, is significant (e.g. the junction highlighted on Fig. 17). Deviant as these junctions' positions may seem, their chord lengths still blend in seamlessly with the other values of the correspondent groups. Thus, we presume that their positions are mainly due to the same prefabrication issue we already mentioned: the quite irregular initial geometry is the reason why in certain points the same values, which fit perfectly in other places, can not be placed between the points which were constructed previously.

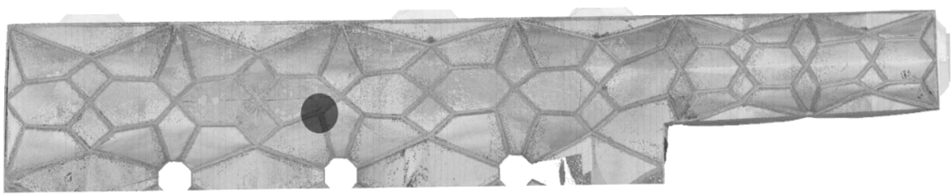

Fig. 17. An example of a highly deviantly placed junction point. The authors' work

4. Discussion

In the previous chapter, we detailed the geometric features of the net vault system of the Church on the Hill of Sighişoara and highlighted the regularities in the system, which led to our conclusion about the methods used to construct the vaults' spatial geometry.

We found that the principle behind the seemingly chaotic vault system is the strictly constructed views of the vaults projected to the short walls, in which the regularities of the longitudinal sections and the plans are interweaving. This construction works with the divisions of quadrate nets based on simple ratios. Moreover, the cross sections' construction also establishes a close relation between the three naves' net vaults, however different their rib pattern may seem. It is worth accentuating that in this guiding principle the underlying idea of division, which is claimed to be the basic Gothic geometric method, is combined with addition, which is generally held as the main Romanesque geometric characteristic. The latter appears in the idea that even though geometric ratio provides a certain degree of connection between the vaults of the aisles and that of the main nave, they are not deduced from the same quadrate net, thus keeping the three parts of the same united hall church space still slightly separated. We think that this latter intention can be parallel to the idea that the three different net vaults of the three naves have different rib patterns, expressing their separation, yet the patterns can be associated with each other, with simple mirroring and the addition of some transversal and longitudinal ribs, as presented on Fig. 18.

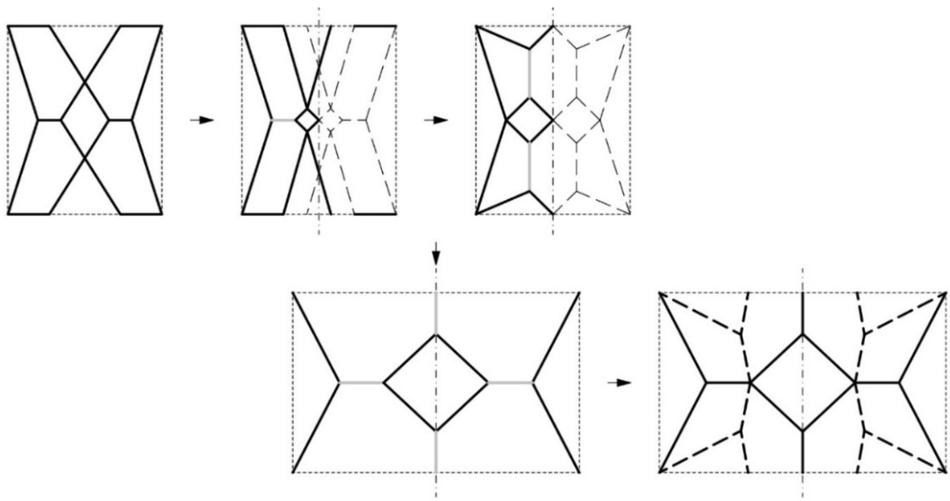

Fig. 18. The geometric connection of the three different plan types. The authors' work

We presume, that the reason to use the cross-section view as the guiding principle instead of the plan view, is the fact we highlighted in the Literature framework chapter, namely that the construction methods based on the plan view suggest a plan of regular borderlines. In this case, however, the given space borders were highly irregular, thus using the plan as the starting point of the vault system's construction would have accentuated these irregularities even more, while the idea actually applied helps to mask the inequalities as best as it seemed possible to the master builder of the Gothic times. Besides the aesthetic considerations, the applied solution could lead to practical benefits as well. The lines in the three-dimensional space (which were described in our analysis with their plan and side views) were supposedly not only theoretical construction lines but also the temporary supporting structure during the building works of the vault system (e. g. main beams supporting the individual rib's centrings). Applying the hereby reconstructed method, these lines were straight, whereas starting the construction from the plan view, they would have been fractioned. This would have resulted in ribs being of different chord lengths, even among the ribs of the same function of the system. Although this would not result in an unsurmountable difficulty, the prefabrication of the rib elements of different lengths would have required significantly more geometric construction work.

Thus, the systematic approach in the seemingly chaotic vaulting system is very much present, in a form which seemed the most practical and sensible to the master and the strict geometric principles actually helped to overcome the difficulties of the construction caused by the highly irregular boundary conditions remaining from the previous building periods.

5. Conclusion

The direct result of our analysis carried out on the net nave vaults of the Church on the Hill of Sighişoara is the description of the geometric regularities in the vault system, and based on those, the deduction of the construction ideas used to create the three-nave system.

We identified the regular geometric features in the seemingly geometric system, such as the straight lines on which the junction points of the ribs fall and the chord length groups characterising the ribs of the same function in the system.

We also found the strict quadrate net construction of the cross-sections based on simple ratios and the combination of the principles of division and addition, which proved to be the underlying idea of the whole net vault system's geometry, which ensures the theoretic and technological relation (and separation) of the three net vaults of significantly different rib patterns.

Apart from the direct results, we think that our most important finding is that even in the case of this prominent town church with a direct architectural connection to the guild of Landshut, well-known for its late Gothic masonry architecture,[51] the main construction principle (the starting point of the construction) proved to be contradictory to the generally accepted ideas of the technical literature due to the constraint of the former building periods' remains. The other question of great importance discussed in the present article is that the main geometric principle predominating the late Gothic net vaults cannot necessarily be found in the pattern of the rib system's plan. According to our hypothesis, these results open up new questions to be examined for future research about the late Gothic monuments of Europe.

[51] Hermann Fabini, and Alida Fabini, *Kirchenburgen in Siebenbürgen* (Leipzig: Koehler&Amelang, 1985), 83; Szőke, "A Wechselberger-Harperger," 204.

6. Acknowledgements

We are grateful to László Daragó DLA for his advice and guidance during the research, to Fanni Budaházi for her valuable help in creating the point cloud of the Church on the Hill of Sighișoara and to Krisztina Fehér PhD for the photos taken in the building. We are also grateful to the organisers and participants of the Summer University on Monument Preservation in Băgaciu/Szászbogács of the year 2022 since the survey of the church was possible within the programs of this event.

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Table 1. The individual rib data of the vault system. *The Sign of the rib* column refers to Fig. 15 and the *Chord length type* column refers to Fig. 16.

Sign of the rib	Chord length [m]	Radius of the curvature [m]	Chord length type
I.1.-A1.1	3.83	4.71	wall arch
I.1.-A1.2	4.02	5.10	wall arch
I.2.-A1.1	3.95	4.73	wall arch
I.2.-A1.2	3.95	4.69	wall arch
I.3.-A1.1	4.10	4.68	wall arch
I.3.-A1.2	4.00	4.58	wall arch
I.4.-A1.1	3.98	4.47	wall arch
I.4.-A1.2	4.03	5.23	wall arch
I.5.-A1.1	3.45	4.74	wall arch
I.5.-A1.2	3.50	7.42	wall arch
I.5.-A2.1	3.95	breakage in the arch	wall arch
I.5.-A2.2	3.65	6.36	wall arch
I.6.-A1.1	3.21	4.52	wall arch
I.6.-A1.2	3.10	4.18	wall arch
I.6.-A2.1	3.23	5.97	wall arch
I.6.-A2.2	3.10	11.13	wall arch
I.1.-B1.1	4.18	4.86	a
I.1.-B1.2	4.20	4.80	a
I.1.-B2.1	4.29	4.44	a
I.1.-B2.2	4.07	4.70	b
I.2.-B1.1	4.33	4.52	a
I.2.-B1.2	4.23	4.57	a
I.2.-B2.1	3.90	4.68	b
I.2.-B2.2	4.08	4.86	b
I.3.-B1.1	4.42	4.65	a
I.3.-B1.2	4.59	4.65	-
I.3.-B2.1	4.16	4.92	a
I.3.-B2.2	4.22	4.73	a
I.4.-B1.1	4.16	4.66	a
I.4.-B1.2	4.35	4.84	a
I.4.-B2.1	3.68	4.78	c
I.4.-B2.2	3.79	4.93	c
I.5.-B1.1	3.84	4.68	c
I.5.-B1.2	3.84	4.51	c
I.5.-B2.1	3.41	4.51	d
I.5.-B2.2	3.81	4.83	c
I.6.-B1.1	3.57	4.24	d
I.6.-B1.2	3.45	4.56	d

Sign of the rib	Chord length [m]	Radius of the curvature [m]	Chord length type
I.6.-B2.1	3.58	4.27	d
I.6.-B2.2	3.52	4.54	d
I.1.-C1.1	2.80	7.50	e
I.1.-C1.2	3.16	4.54	e
I.1.-C2.1	2.84	7.32	f
I.1.-C2.2	2.59	4.40	e
I.2.-C2.1	3.23	4.16	f
I.2.-C2.2	2.35	4.35	g
I.3.-C2.1	3.23	4.07	f
I.3.-C2.2	2.34	4.45	g
I.4.-C2.1	3.26	4.10	f
I.4.-C2.2	3.28	6.81	f
I.5.-C2.1	3.10	6.91	f
I.5.-C2.2	2.90	6.60	e
I.6.-C2.1	3.24	4.67	f
I.6.-C2.2	2.92	4.47	e
I.1.-D1	0.73	non measurable	h
I.1.-D2	0.73	non measurable	h
I.2.-D1	0.76	non measurable	h
I.2.-D2	0.79	non measurable	h
I.3.-D1	0.73	non measurable	h
I.3.-D2	0.76	non measurable	h
I.4.-D1	0.75	non measurable	h
I.4.-D2	0.79	non measurable	h
I.5.-D1	0.77	non measurable	h
I.5.-D2	0.79	non measurable	h
I.6.-D1	0.72	non measurable	h
I.6.-D2	0.72	non measurable	h
I.1.-E1.1	2.17	6.57	i
I.1.-E1.2	2.01	7.08	i
I.1.-E2.1	2.19	non measurable	i
I.1.E2.2	2.08	6.60	i
I.2.-E1.1	2.13	9.46	i
I.2.-E1.2	2.02	8.19	i
I.2.-E2.1	2.04	7.05	i
I.2.-E2.2	2.07	11.44	i
I.3.-E1.1	2.13	9.15	i
I.3.-E1.2	2.08	8.67	i
I.3.-E2.1	2.29	10.49	i
I.3.-E2.2	2.21	10.10	i

Sign of the rib	Chord length [m]	Radius of the curvature [m]	Chord length type
I.4.-E1.1	1.80	6.08	i
I.4.-E1.2	2.02	11.93	i
I.4.-E2.1	2.05	non measurable	i
I.4.-E2.2	1.83	non measurable	i
I.5.-E1.1	1.73	8.69	j
I.5.-E1.2	1.69	9.00	i
I.5.-E2.1	2.02	non measurable	j
I.5.-E2.2	1.77	non measurable	j
I.6.-E1.1	1.53	7.62	k
I.6.-E1.2	1.47	non measurable	k
I.6.-E2.1	1.73	non measurable	k
I.6.-E2.2	1.50	non measurable	m
I.1.-F1.1	1.47	6.87	k
I.1.-F1.2	1.58	6.79	k
I.1.-F2.1	1.61	6.51	k
I.1.-F2.2	1.50	7.60	k
I.2.-F1.1	1.42	6.96	k
I.2.-F1.2	1.59	6.98	k
I.2.-F2.1	1.42	7.63	k
I.2.-F2.2	1.45	8.39	k
I.3.-F1.1	1.55	8.35	k
I.3.-F1.2	1.54	9.49	k
I.3.-F2.1	1.54	6.93	k
I.3.-F2.2	1.62	7.17	k
I.4.-F1.1	1.43	6.29	k
I.4.-F1.2	1.29	6.72	k
I.4.-F2.1	1.49	non measurable	k
I.4.-F2.2	1.44	non measurable	k
I.5.-F1.1	1.49	7.36	k
I.5.-F1.2	1.54	non measurable	k
I.5.-F2.1	1.43	non measurable	k
I.5.-F2.2	1.50	non measurable	k
I.6.-F1.1	1.46	8.95	k
I.6.-F1.2	1.42	9.32	k
I.6.-F2.1	1.59	non measurable	k
I.6.-F2.2	1.40	6.80	k
I.1.-G1	1.30	7.50	e
I.1.-G2	1.43	7.32	e
I.6.-G1	0.90	4.67	l
I.6.-G2	0.95	4.47	l

Sign of the rib	**Chord length [m]**	**Radius of the curvature [m]**	**Chord length type**
II.1.-B1.1	4,64	4,7	-
II.1.-B1.2	4,41	5,5	o
II.1.-B2.1	4,43	4,45	o
II.1.-B2.2	4,42	4,68	o
II.2.-B1.1	4,35	4,86	o
II.2.-B1.2	4,34	4,68	o
II.2.-B2.1	4,1	4,6	q
II.2.-B2.2	4,28	4,68	o
II.3.-B1.1	4,34	4,95	o
II.3.-B1.2	4,37	4,94	o
II.3.-B2.1	4,38	4,76	o
II.3.-B2.2	4,32	4,75	o
II.4.-B1.1	4,12	4,6	q
II.4.-B1.2	4,54	4,92	-
II.4.-B2.1	4,05	4,63	q
II.4.-B2.2	4,315	4,52	o
II.1.-C1.1	5,18	6,48	wall arch
II.1.-C2.1	5,25	6,8	wall arch
II.4.-C1.1	5,51	7,78	wall arch
II.4.-C2.1	5,35	7,45	wall arch
II.1.-D1	1,65	non-measurable	n
II.1.-D2	1,66	non-measurable	n
II.2.-D1	1,65	non-measurable	n
II.2.-D2	1,77	non-measurable	n
II.3.-D1	1,73	non-measurable	n
II.3.-D2	1,78	12,11	n
II.4.-D1	1,74	non-measurable	n
II.4.-D2	1,7	non-measurable	n
II.1.-E1.1	4,82	5,05	-
II.1.-E1.2	4,105	4,76	q
II.1.-E2.1	4,89	5,13	-
II.1.-E2.2	4,48	4,24	o
II.2.-E1.1	4,43	4,155	o
II.2.-E1.2	4,33	4,08	p
II.2.-E2.1	4,23	5,5	p
II.2.-E2.2	4,22	4,08	p
II.3.-E1.1	4,34	4,88	p
II.3.-E1.2	4,16	4,9	p
II.3.-E2.1	4,31	4,53	p
II.3.-E2.2	4,19	4,51	p

Sign of the rib	Chord length [m]	Radius of the curvature [m]	Chord length type
II.4.-E1.1	4,2	5,6	p
II.4.-E1.2	5,26	7,16	-
II.4.-E2.1	4,215	4,82	p
II.4.-E2.2	4,95	5,6	-
II.1.-F1.1	1,5	non-measurable	m
II.1.-F1.2	1,41	non-measurable	m
II.1.-F2.1	1,27	non-measurable	m
II.1.-F2.2	1,46	non-measurable	m
II.2.-F1.1	1,5	non-measurable	m
II.2.-F1.2	1,43	non-measurable	m
II.2.-F2.1	1,36	non-measurable	m
II.2.-F2.2	1,38	non-measurable	m
II.3.-F1.1	1,62	non-measurable	n
II.3.-F1.2	1,64	non-measurable	n
II.3.-F2.1	1,46	non-measurable	m
II.3.-F2.2	1,47	non-measurable	m
II.4.-F1.1	1,375	non-measurable	m
II.4.-F1.2	1,14	non-measurable	-
II.4.-F2.1	1,35	non-measurable	m
II.4.-F2.2	1,08	non-measurable	-
II.1.-G1.1	2,28	non-measurable	r
II.1.-G1.2	2,34	8,37	r
II.1.-G2.1	2,29	8,76	r
II.1.-G2.2	2,23	non-measurable	r
II.2.-G1.1	2,22	non-measurable	r
II.2.-G1.2	2,38	non-measurable	r
II.2.-G2.1	2,36	non-measurable	r
II.2.-G2.2	2,25	non-measurable	r
II.3.-G1.1	2,28	non-measurable	r
II.3.-G1.2	2,26	non-measurable	r
II.3.-G2.1	2,36	non-measurable	r
II.3.-G2.2	2,23	non-measurable	r
II.4.-G1.1	2,41	non-measurable	r
II.4.-G1.2	2,265	non-measurable	r
II.4.-G2.1	2,32	non-measurable	r
II.4.-G2.2	2,31	non-measurable	r
II.1.-H1	1,89	non-measurable	l
II.1.-H2	1,85	non-measurable	l
II.4.-H1	1,795	non-measurable	l
II.4.-H2	1,91	non-measurable	l

Sign of the rib	Chord length [m]	Radius of the curvature [m]	Chord length type
II.1.-J1	1,52	non-measurable	m
II.1.-J2	1,51	non-measurable	m
II.2.-J1	1,47	non-measurable	m
II.2.-J2	1,48	non-measurable	m
II.3.-J1	1,66	non-measurable	n
II.3.-J2	1,49	non-measurable	m
II.4.-J1	1,26	non-measurable	-
II.4.-J2	1,29	non-measurable	-
II.1.-L1	1,61	non-measurable	n
II.1.-L2	1,61	non-measurable	n
II.2.-L1.1	1,6	non-measurable	n
II.2.-L1.2	1,64	non-measurable	n
II.2.-L2.1	1,655	non-measurable	n
II.2.-L2.2	1,76	non-measurable	n
II.3.-L1.1	1,72	non-measurable	n
II.3.-L1.2	1,43	non-measurable	m
II.3.-L2.1	1,78	non-measurable	n
II.3.-L2.2	1,64	non-measurable	n
II.4.-L1	1,44	non-measurable	m
II.4.-L2	1,43	non-measurable	m
II.1.-M1	1,83	0,105	s
II.1.-M2	1,78	0,07	s
II.2.-M1	1,7	0,11	s
II.2.-M2	1,8	0,87	s
II.3.-M1	1,74	0,05	s
II.3.-M2	1,78	0,08	s
III.1.-A2.1	3,775	4,54	wall arch
III.1.-A2.2	3,89	4,4	wall arch
III.2.-A2.1	3,89	4,76	wall arch
III.2.-A2.2	3,78	4,34	wall arch
III.3.-A2.1	4,07	4,89	wall arch
III.3.-A2.2	3,98	4,6	wall arch
III.4.-A2.1	3,94	4,49	wall arch
III.4.-A2.2	3,88	4,67	wall arch
III.5.-A1.1	2,92	7,31	wall arch
III.5.-A1.2	2,99	8,68	wall arch
III.6.-A1.1	2,97	8,44	wall arch
III.6.-A1.2	3,15	7,77	wall arch
III.6.-A2.1	2,97	7,06	wall arch
III.6.-A2.2	3,13	4,44	wall arch

Sign of the rib	Chord length [m]	Radius of the curvature [m]	Chord length type
III.1.-B1.1	5,14	4,42	-
III.1.-B1.2	4,17	4,7	v
III.1.-B2.1	4,24	4,83	v
III.1.-B2.2	4,41	4,94	v
III.2.-B1.1	4,15	4,86	v
III.2.-B1.2	4,16	4,77	v
III.2.-B2.1	4,24	4,37	v
III.2.-B2.2	4,32	4,57	v
III.3.-B1.1	4,25	4,54	v
III.3.-B1.2	4,2	4,83	v
III.3.-B2.1	4,4	4,7	v
III.3.-B2.2	4,13	4,62	v
III.4.-B1.1	3,81	4,86	-
III.4.-B1.2	4,105	5,02	v
III.4.-B2.1	4,24	4,53	v
III.4.-B2.2	4,15	4,63	v
III.5.-B1.1	3,17	4,64	ad
III.5.-B1.2	3,19	4,72	ad
III.5.-B2.1	3,33	4,33	ad
III.5.-B2.2	3,2	4,26	ad
III.6.-B1.1	3,23	4,41	ad
III.6.-B1.2	3,32	4,65	ad
III.6.-B2.1	3,22	4,515	ad
III.6.-B2.2	3,4	4,325	ad
III.1.-C1.1	4,17	6,28	wall arch
III.1.-C1.2	3,82	7,06	wall arch
III.1.-D1.1	1,15	6,04	z
III.1.-D1.2	1,11	4,02	z
III.1.-D1.3	1,15	5,175	z
III.1.-D1.4	1,2	4,05	z
III.1.-D2.1	1,1	5,34	z
III.1.-D2.2	1,03	4,59	z
III.1.-D2.3	1,16	7,08	z
III.1.-D2.4	1,085	10,165	z
III.2.-D1.1	1,1	5,59	z
III.2.-D1.2	1,2	4,88	z
III.2.-D1.3	1,105	4,25	z
III.2.-D1.4	1,13	6,23	z
III.2.-D2.1	1,14	5,37	z
III.2.-D2.2	1,12	14,88	z

Sign of the rib	Chord length [m]	Radius of the curvature [m]	Chord length type
III.2.-D2.3	1,16	6,265	z
III.2.-D2.4	1,02	4,24	z
III.3.-D1.1	1,22	5,17	z
III.3.-D1.2	1,22	8	z
III.3.-D1.3	1,27	8,12	z
III.3.-D1.4	1,13	6,29	z
III.3.-D2.1	1,44	8,07	z
III.3.-D2.2	1,09	8,5	z
III.3.-D2.3	1,16	5,78	z
III.3.-D2.4	1,13	6,75	z
III.4.-D1.1	0,87	6,06	aa
III.4.-D1.2	1,23	5,855	z
III.4.-D1.3	1,165	7,45	z
III.4.-D1.4	1,03	5,01	z
III.4.-D2.1	1,185	5,22	z
III.4.-D2.2	1,21	5,22	z
III.4.-D2.3	0,93	7,7	z
III.4.-D2.4	1,125	6,255	z
III.5.-D1.1	0,73	non measurable	aa
III.5.-D1.2	0,795	non measurable	aa
III.5.-D1.3	1,015	-	z
III.5.-D1.4	0,94	-	z
III.5.-D2.1	0,78	-	aa
III.5.-D2.2	0,7	-	aa
III.5.-D2.3	0,97	-	z
III.5.-D2.4	0,925	-	z
III.6.-D1.1	0,76	-	aa
III.6.-D1.2	0,64	-	aa
III.6.-D1.3	0,88	-	aa
III.6.-D1.4	0,97	-	z
III.6.-D2.1	0,53	-	-
III.6.-D2.2	0,66	-	aa
III.6.-D2.3	1,06	-	z
III.6.-D2.4	0,88	-	aa
III.1.-E1.1	1,54	8,84	x
III.1.-E1.2	1,52	5,51	x
III.1.-E2.1	1,45	5,185	x
III.1.-E2.2	1,56	9,59	x
III.2.-E1.1	1,31	7,12	x
III.2.-E1.2	1,43	8,82	x

Sign of the rib	Chord length [m]	Radius of the curvature [m]	Chord length type
III.2.-E2.1	1,44	6,89	x
III.2.-E2.2	1,49	6,81	x
III.3.-E1.1	1,41	6,575	x
III.3.-E1.2	1,49	6,77	x
III.3.-E2.1	1,39	10,05	x
III.3.-E2.2	1,58	8,045	x
III.4.-E1.1	1,34	7,54	x
III.4.-E1.2	1,98	6,5	the connection between the eastern and western system
III.4.-E2.1	1,49	8,58	x
III.4.-E2.2	1,76	8,65	the connection between the eastern and western system
III.5.-E1.1	1,19	5,74	ac
III.5.-E1.2	1,18	11,55	ac
III.5.-E2.1	1,07	6,4	ac
III.5.-E2.2	1,21	12,86	ac
III.6.-E1.1	1,3	5,94	ac
III..-E1.2	1,275	8,88	ac
III.6.-E2.1	1,34	8,16	ac
III.6.-E2.2	1,27	6,38	ac
III.1.-F1.1	3,96	3,67/4,075	u
III.1.-F1.2	3,31	5,03	y
III.1.-F2.1	3,78	5,62	u
III.1.-F2.2	4,09	4,86	u
III.2.-F1.1	3,16	4,86	y
III.2.-F1.2	3,29	4,62	y
III.2.-F2.1	4,03	4,9	u
III.2.-F2.2	4,08	4,72	u
III.3.-F1.1	3,53	5,4	y
III.3.-F1.2	3,43	4,84	y
III.3.-F2.1	4,11	5,5	u
III.3.-F2.2	3,84	4,71	u
III.4.-F1.1	3,37	5,03	y
III.4.-F1.2	2,63	7,32	ab
III.4.-F2.1	3,84	5,79	u
III.4.-F2.2	2,87	11,25	ab
III.5.-F1.1	2,58	8,58	ab
III.5.-F1.2	2,74	7,635	ab

Sign of the rib	Chord length [m]	Radius of the curvature [m]	Chord length type
III.5.-F2.1	2,83	8,18	ab
III.5.-F2.2	2,69	6,26	ab
III.6.-F1.1	2,61	7,54	ab
III.6.-F1.2	2,72	7,36	ab
III.6.-F2.1	2,685	8,26	ab
III.6.-F2.2	2,7	8,27	ab
III.1.-G1.1	1,13	4	w
III.1.-G1.2	0,99	6,17	w
III.1.-G2.1	0,95	5,4	w
III.1.-G2.2	1	6,15	w
III.2.-G1.1	1,15	8	w
III.2.-G1.2	0,945	4,28	w
III.2.-G2.1	0,97	7,67	w
III.2.-G2.2	1,03	4,79	w
III.3.-G1.1	1	5,24	w
III.3.-G1.2	0,97	5,95	w
III.3.-G2.1	1,11	7,55	w
III.3.-G2.2	0,995	8,96	w
III.4.-G1.1	1,2	7,42	w
III.4.-G1.2	1,16	5,75	w
III.4.-G2.1	1,035	9,89	w
III.4.-G2.2	1,07	5,08	w
III.5.-G1.1	1,11	6,78	w
III.5.-G1.2	1,06	6,24	w
III.5.-G2.1	1,14	non measurable	w
III.5.-G2.2	1,09	5,9	w
III.6.-G1.1	1	4,4	w
III.6.-G1.2	1,04	4,48	w
III.6.-G2.1	1	7,8	w
III.6.-G2.2	1,04	-	w

Table 2. The average and dispersion values of the different rib type groups.

Sign of the rib type	Type average	Type dispersion
a	4,25	0,09
b	4,02	0,10
c	3,79	0,07
d	3,51	0,07
e	2,87	0,06
f	3,21	0,06
g	2,35	0,00
h	0,75	0,03
i	2,07	0,12
j	1,73	0,03
k	1,49	0,08
l	1,86	0,05
m	1,44	0,06
n	1,68	0,06
o	4,37	0,06
p	4,24	0,07
q	4,09	0,03
r	2,30	0,06
s	1,77	0,05
t	1,28	0,02
u	3,97	0,13
v	4,23	0,10
w	1,05	0,07
x	1,46	0,08
y	3,35	0,13
z	1,12	0,10
aa	0,77	0,09
ab	2,71	0,09
ac	1,23	0,09
ad	3,26	0,08