

# ON THE ABSENCE OF SUPERCONDUCTIVITY IN GAPPED GRAPHENE SYSTEMS

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**ABSTRACT.** We analyzed the possibility of the superconducting state in the case of gapped graphene systems. For the electron excitations we consider the case of massive gapped spectrum and the case of massless gapped spectrum. Using realistic parameters we showed that the superconducting state is absent.

**Keywords:** *Superconductivity, Graphene, Massive gapped spectrum, Massless gapped spectrum.*

Graphene is the first two dimensional crystal observed in nature that possesses remarkable physical properties [1]. Due to its band structure graphene is a zero density of states semimetal at the Fermi energy. The low energy excitations are characterized by linear dispersion of the quasiparticles in the vicinity of the Fermi points. Pristine graphene is less useful for practical applications because of its low carrier density and zero band gap. On the other hand, many electronic applications require the presence of an energy gap between the bands. Several experimental measurements [2, 3] reveal the presence of an energy gap  $\Delta_g$  in the quasiparticle spectrum of graphene. The nature of the gap was attributed to the effect of the substrate. The gapped energy spectrum (massive gapped spectrum) was considered in order to

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analyze the thermoelectric power in graphene [4], and the phenomenological massless gapped spectrum [5] was introduced to reconcile the gapped nature of the energy spectrum with several spectroscopic measurements. Other physical properties of gapped graphene systems were analyzed in Refs.[6, 7]. The presence of the gap  $\Delta_g$  changes the electronic density of states and affect many of the physical properties of graphene. Recent experiments reveal that a doped bilayer graphene twisted around a certain angle becomes superconducting [8]. A weak-coupling theory of superconductivity for electrons in graphene was proposed in Ref.[9] and extended in Ref.[10]. Here was shown that the superconducting state, for Dirac electrons, occur if the attractive electron-electron interaction  $\lambda$  exceeds a critical value  $\lambda_c$ . Unfortunately, taking realistic parameters,  $\lambda_c \cong 20 \cdot \lambda$ , which shows that pristine graphene cannot reach the superconducting state. An opposite behaviour is obtained in the case of doped graphene where the superconducting state occur, and the superconducting parameters (superconducting energy-gap, critical temperature, Gelikman-Kresin ratio, etc.) [10-13] can be calculated.

In this paper we will analyze the possibility of the superconducting state in gapped graphene systems for both, the massive and the massless gapped spectrum. We will adopt the mean-field scenario where the superconducting gap equation has the self-consistent form [10]

$$\Delta = 4\lambda \int_0^{k_c} \frac{d^2k}{(2\pi)^2} \cdot \frac{\Delta}{E_k} \cdot \tanh\left(\frac{E_k}{2k_B T}\right) \quad (1)$$

$\Delta$ - is the superconducting energy gap,  $\lambda$ - the electron-electron coupling factor,  $T$ - the temperature, and  $k_c$ - the cut-off momentum. Here

$$E_k = \sqrt{\varepsilon_k^2 + \Delta^2} \quad (2)$$

For the case of massive gapped system one has

$$\varepsilon_k = \pm \sqrt{\hbar^2 v_F^2 k^2 + \Delta_g^2} \quad (3)$$

with  $v_F$  - the Fermi velocity. The zero temperature superconducting energy gap ( $\Delta \rightarrow \Delta_0$ , for  $T \rightarrow 0$ ), will be calculated using eq.(1) with  $\tanh(E_k / 2k_B T) \rightarrow 0$ . We obtain

$$1 = \frac{2\lambda}{\pi} \int_0^{k_c} \frac{dk \cdot k}{\sqrt{\hbar^2 v_F^2 k^2 + \Delta_g^2 + \Delta_0^2}} \quad (4)$$

Introducing:  $x = \hbar^2 v_F^2 k^2$ , and evaluating the integral, we have

$$\frac{\pi \hbar^2 v_F^2}{2\lambda} = \sqrt{\varepsilon_c^2 + \Delta_g^2 + \Delta_0^2} - \sqrt{\Delta_g^2 + \Delta_0^2} \quad (5)$$

Here:  $\varepsilon_c = \hbar v_F k_c$ . We define now  $\lambda_c = \pi \hbar^2 v_F^2 / 2\varepsilon_c$  - the critical coupling. The equation for the superconducting gap  $\Delta_0$  will be

$$\sqrt{\Delta_g^2 + \Delta_0^2} = \varepsilon_c \cdot \frac{\lambda^2 - \lambda_c^2}{2\lambda\lambda_c} \quad (6)$$

For realistic parameters of graphene it was shown [10] that  $\lambda_c \approx 20 \cdot \lambda$ . One conclude, using eq.(6), that the superconducting state for graphene with massive gapped spectrum is not possible.

For the case of massless gapped spectrum

$$\varepsilon_k = \pm(\hbar v_F k + \Delta_g) \quad (7)$$

and using eq.(1), the zero temperature superconducting energy gap equation will be

$$1 = \frac{2\lambda}{\pi} \int_0^{k_c} \frac{dk \cdot k}{\sqrt{(\hbar v_F k + \Delta_g)^2 + \Delta_0^2}} \quad (8)$$

With the new variable:  $y = \hbar v_F k + \Delta_g$ , splitting the integral above in two contributions, and after evaluating the integrals, we obtain

$$\varepsilon_c \cdot \frac{\lambda_c}{\lambda} = \sqrt{(\varepsilon_c + \Delta_g)^2 + \Delta_0^2} - \sqrt{\Delta_g^2 + \Delta_0^2} - \Delta_g \cdot \ln \left( \frac{\varepsilon_c + \Delta_g + \sqrt{(\varepsilon_c + \Delta_g)^2 + \Delta_0^2}}{\Delta_g + \sqrt{\Delta_g^2 + \Delta_0^2}} \right) \quad (9)$$

Due to the large value of the ratio  $\lambda_c / \lambda$  one can observe that the right hand side of eq.(9) cannot reach the value given by the left hand side. To give a more transparent result we will consider the following case:  $\varepsilon_c, \Delta_g \gg \Delta_0$ . In this case one can expand the right hand side in  $\Delta_0$  to obtain

$$\varepsilon_c \left( \frac{\lambda_c}{\lambda} - 1 \right) + \Delta_g \cdot \ln \left( 1 + \frac{\varepsilon_c}{\Delta_g} \right) \cong - \frac{\Delta_0^2}{4\Delta_g} \left( \frac{\varepsilon_c}{\varepsilon_c + \Delta_g} \right)^2 \quad (10)$$

Due to the fact that:  $\lambda_c > \lambda$  ,  $\Delta_g > 0$  ,  $\varepsilon_c > 0$  , one can observe that the superconducting state is not possible nor in the case of massless gapped spectrum.

Our simple model show the absence of superconductivity in the case of graphene, in both the massive and the massless gapped spectrum models, even an attractive electron-electron interaction is present.

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